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Research article

Some Topological Properties of Minkowski Difference in Euclidean Spacess

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Abstract

This paper examines important topological properties of Minkowski operations, namely, the preservation of density by the Minkowski difference and its continuity. We obtain a condition under which the density property of sets is preserved by the Minkowski difference. Furthermore, we establish that the Minkowski difference operation is continuous.

Keywords: *topological vector space; Minkowski difference; support functions; class of compact convex sets; continuous mapping.*

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1. Introduction

Topological vector spaces are among the fundamental objects of functional analysis and modern geometry. These spaces arise from considering algebraic and topological structures together on a single set, allowing the study of topological concepts such as limits, continuity, convergence, and compactness alongside linear operations. It is precisely this aspect that makes topological vector spaces an important tool for solving problems arising in many mathematical fields, including geometry, differential equations, optimal control, variational calculus, and convex analysis.

Topological vector spaces are extensively studied in many works. In particular, modern references such as Bogachev V.I., Smolyanov O.G. [1] and Narici L., Beckenstein E. [2] are useful sources on the general theory of topological vector spaces. These works present the definition of a topological vector space, the requirements imposed on structures that turn a linear space into a topological space, and important examples. They also cover fundamental concepts such as duality, topologies on dual spaces, continuous mappings, the Hahn-Banach theorem, and barrelled spaces.

Among the algebraic operations defined on sets in topological vector spaces, the Minkowski sum and Minkowski difference are of particular importance and are widely used in convex analysis, geometric optimization, and differential game theory. The Minkowski sum has been considered as a primary object in many studies because it preserves convexity, has a natural connection with support functions, and exhibits good compatibility with topological properties. The geometric and analytical properties of this operation are extensively covered in the works of Rockafellar R. [3], Schneider R. [4], Dvurechensky P.E., Ivanov G.E.[5] .

A number of recent studies have also reported results concerning Minkowski operations. In particular, in [12, 13], the Hukuhara difference, which is a special case of the Minkowski difference, is investigated for compact convex sets, and its various properties are studied. Furthermore, [12] work presents results on the Hausdorff distance between convex sets, which will be used in the subsequent sections of this paper.

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In paper [14], another inverse operation to the Minkowski sum, called the extended set difference, is introduced for compact convex sets. This difference is a generalization of both the Hukuhara difference and the generalized Hukuhara difference. The proposed set difference is defined in terms of the Hausdorff metric. However, unlike the geometric Minkowski difference studied in this paper, it is based on an optimization framework and guarantees the existence of the difference for every pair of compact convex sets.

The Minkowski difference has found wide applications in solving problems in robotics and autonomous navigation [15], robot motion planning [16], optimal control [17] and computational geometry [18]. At present, research in these areas remains highly relevant and continues to attract considerable attention.

2. Preliminaries and Definitions

Definition 2.1. Let non-empty sets $X, Y \subset \mathbb{R}^n$ be given in $\mathbb{R}^n$. The geometric (or Minkowski) sum and difference of the sets X and Y are defined, respectively, as the following sets:

$$X + Y = \{x + y : x \in X, y \in Y\}, \qquad X \dot{-} Y = \{z \in \mathbb{R}^n : z + Y \subset X\}. \tag{2.1}$$

Definition 2.2. The product of a set X by a scalar λ is defined as the following set:

$$\lambda X = \{\lambda x : x \in X\}. \tag{2.2}$$

According to the above definition, for an arbitrary vector $a \in \mathbb{R}^n$ and a non-empty set $X \subset \mathbb{R}^n$, the Minkowski sum is understood as

$$a + X = \{a + x : x \in X\}. \tag{2.3}$$

It follows directly from the definition that the Minkowski difference $X \dot{-} Y$ is the intersection of the geometric loci of parallel translations of the set X by vectors $d \in -Y$ [6]; that is,

$$X \dot{-} Y = \bigcap_{d \in -Y} (X + d). \tag{2.4}$$

The properties of Minkowski operations, geometric methods for computing them, and various applications are studied in [7], [8], [9]. In particular, in [7], methods for finding the Minkowski difference for all cases of sets consisting of regions bounded by a triangle in the plane are proposed. In [8], the method for finding the Minkowski difference is generalized to arbitrary sets in space.

3. Main Results

The methods and examples presented in research on the topic show that Minkowski operations are easier and more efficient to perform in lower-dimensional spaces (e.g., in $\mathbb{R}^2$). However, solving practical problems often requires working with sets given in higher-dimensional spaces. Under such conditions, it is convenient to decompose the Cartesian product of a given set into lower-dimensional subsets and work with them. The following statement can be proved by direct verification.

Proposition 3.1. *For non-empty sets A, B, C and D in $\mathbb{R}^n$, the following equality holds:*

$$(A \times B) + (C \times D) = (A + C) \times (B + D). \tag{3.1}$$

Here, $A \times B$ denotes the Cartesian product of sets.

Equality (3.1) can also be written for the Minkowski difference. The following statement proves this equality.

Proposition 3.2. *For non-empty sets A, B, C and D in $\mathbb{R}^n$, if the Minkowski differences $A\dot{-}B$ and $C\dot{-}D$ are non-empty, then the following equality holds:*

$$(A \times C)\dot{-}(B \times D) = (A\dot{-}B) \times (C\dot{-}D). \tag{3.2}$$

Proof. Suppose $(x, y) \in (A \times C)\dot{-}(B \times D)$. Then, by the definition of the Minkowski difference, we have $(x, y) + (B \times D) \subset (A \times C)$. According to Proposition 3.1, for any elements $b \in B$ and $d \in D$, we have $(x + b, y + d) \in (A \times C)$. This implies $x + b \in A$ and $y + d \in C$. Since these relations hold for all $b \in B$ and $d \in D$, it follows that $x \in A\dot{-}B$ and $y \in C\dot{-}D$. Hence, $(x, y) \in (A\dot{-}B) \times (C\dot{-}D)$.

Now, take an arbitrary element $(x, y) \in (A\dot{-}B) \times (C\dot{-}D)$. By the definition of the Cartesian product, $x \in A\dot{-}B$ and $y \in C\dot{-}D$. By the definition of the Minkowski difference, for any $b \in B$ and $d \in D$, we have $x + b \in A$ and $y + d \in C$. From these, we obtain $(x + b, y + d) \in (A \times C)$. By the definition of the Cartesian product and proposition 3.1, we have:

$$(x, y) + (b, d) \in (A \times C),$$

$$(x, y) + (B \times D) \subset (A \times C).$$

The second inclusion implies $(x, y) \in (A \times C)\dot{-}(B \times D)$. This completes the proof of Proposition 3.2. □

The notion of density in topological spaces is one of the important concepts. Since parallel translation of a set by a vector is a homeomorphism, if a set A is dense in $X \subset \mathbb{R}^n$, then for any vector $a \in \mathbb{R}^n$, the set $a + A$ is dense in $a + X$. Given that $A\dot{-}a = A + (-a)$, it follows that if A is dense in X, then for any vector a, the set $A\dot{-}a$ is dense in $X\dot{-}a$.

The property of density is not always preserved under the Minkowski difference operation. The following example shows that when A is dense in X and B is dense in Y, the difference $A\dot{-}B$ is not necessarily dense in $X\dot{-}Y$.

Example 3.1. In $\mathbb{R}$, the set $A = (-1, 2) \setminus \{0\}$ is dense in $X = (-1, 2)$, and the set $B = (0, 1)$ is dense in $Y = (0, 1]$. However, the set $A\dot{-}B = \{-1\} \cup [0, 1]$ is not dense in $X\dot{-}Y = [-1, 1)$.

Thus, additional conditions are required to preserve the density property. Suppose that for sets A, B, X, Y, the Minkowski differences $A\dot{-}B$ and $X\dot{-}Y$ are non-empty.

Theorem 3.1. *If set A is dense in X, set B is dense in Y and $\operatorname{int}((X \setminus A) + (-B)) = \emptyset$, then the set $A\dot{-}B$ is dense in $X\dot{-}Y$.*

To prove this theorem, we use the following auxiliary lemma.

Lemma 3.1. *For arbitrary non-empty sets $A, B, C \subset \mathbb{R}^n$, the following relation holds:*

$$(A\dot{-}B)\setminus(C\dot{-}B) \subset (A\setminus C) + (-B) \tag{3.3}$$

Proof of Lemma 3.1. Take an arbitrary element x from the set $(A\dot{-}B) \setminus (C\dot{-}B)$. Since $x \in (A\dot{-}B) \setminus (C\dot{-}B)$, we have $x \in A\dot{-}B$ and $x \notin C\dot{-}B$. By the definition of the Minkowski difference, we can write

$$x + B \subset A \tag{3.4}$$

and

$$x + B \not\subset C. \tag{3.5}$$

From relation (3.5), there exists some $b_0 \in B$ such that $x + b_0 \notin C$. However, relation (3.4) implies that $x + b_0 \in A$. Therefore,

$$x + b_0 \in A \setminus C. \tag{3.6}$$

From relation (3.6), it follows that $x \in (A \setminus C) + (-B)$. Lemma 3.1 is proved.

Proof of Theorem 3.1. According to the assumptions of the theorem, the set B is dense in the set Y. Hence, $B \subset Y$. By a property of the Minkowski difference, we obtain

$$X \dot{-} B \supset X \dot{-} Y.$$

Consequently,

$$(X \dot{-} Y) \setminus (A \dot{-} B) \subset (X \dot{-} B) \setminus (A \dot{-} B). \tag{3.7}$$

By Lemma 3.1,

$$(X \dot{-} B) \setminus (A \dot{-} B) \subset (X \setminus A) + (-B).$$

By the assumptions of the theorem, $\operatorname{int}\big((X \setminus A) + (-B)\big) = \varnothing$. Therefore, the set $(X \dot{-} Y) \setminus (A \dot{-} B)$ also has empty interior. Hence, the set $A \dot{-} B$ is dense in the set $X \dot{-} Y$. This completes the proof of Theorem 3.1.

In the space $\mathbb{R}^n$, the distance between two compact convex subsets X and Y is defined as the smallest value among all quantities r satisfying the conditions [11]

$$X \subset Y + B_r, \qquad Y \subset X + B_r.$$

The distance between X and Y defined in this way is denoted by $h(X, Y)$. Thus, the family of all nonempty compact convex subsets of $\mathbb{R}^n$, denoted by $cc(\mathbb{R}^n)$, forms a metric space.

Lemma 3.2. *The Minkowski difference $A \dot{-} B$ of compact convex sets $A, B \subset \mathbb{R}^n$ is also a compact convex set.*

Proof. According to [11], since A is convex, it follows that $A \dot{-} B$ is also convex. By [6], the Minkowski difference can be represented as

$$A \dot{-} B = \bigcap_{b \in -B} (A + b).$$

Since the Minkowski sum is continuous, the set $A + b$ is compact for every $b \in -B$. By [1], every closed subset of a compact set is compact. Therefore, for any vector $b_0 \in -B$,

$$\bigcap_{b \in -B} (A + b) \subset A + b_0.$$

Because $A + b_0$ is compact and an arbitrary intersection of closed sets is closed, it follows that the difference $A \dot{-} B$ is compact. □

Definition 3.1. The support function of a convex set $X \subset \mathbb{R}^n$ is the function defined for a vector $p \in \mathbb{R}^n$ by

$$s_X(p) = \sup_{x \in X} \langle x, p \rangle.$$

Here, $\langle x, p \rangle$ denotes the scalar product of the vectors x and p.

Lemma 3.3. *Let $X, Y \subset \mathbb{R}^n$ be nonempty compact convex sets such that the Minkowski difference $X \dot{-} Y$ is nonempty. Then the following equality holds:*

$$(X + B_r(0)) \dot{-} Y = (X \dot{-} Y) + B_r(0).$$

Here, $B_r(0)$ denotes the open ball centered at the origin with radius r.

Proof. To establish the inclusion $(X + B_r(0)) \dot{-} Y \subset (X \dot{-} Y) + B_r(0)$, we first prove that

$$(X + \overline{B}_r(0)) \dot{-} Y \subset (X \dot{-} Y) + \overline{B}_r(0).$$

Here, $\overline{B}_r(0)$ denotes the closed ball centered at the origin with radius r.

Assume that $z \in (X + \overline{B}_r(0))\dot{-}Y$. Then, by the definition of the Minkowski difference,

$$z + Y \subset X + \overline{B}_r(0). \tag{3.8}$$

Using support functions, relation (3.8) can be written as

$$\langle p, z\rangle + s_Y(p) \le s_X(p) + s_{\overline{B}_r(0)}(p). \tag{3.9}$$

Since $s_{\overline{B}_r(0)}(p)$ is the support function of the closed bounded ball $\overline{B}_r(0)$, it can be represented in the form $s_{\overline{B}_r(0)}(p) = \max|b| \le r\langle p, b\rangle$. Suppose that for some $b_0 \in \overline{B}_r(0)$, $s_{\overline{B}_r(0)}(p) = \langle p, b_0\rangle$. Then inequality (3.9) becomes

$$\langle p, z\rangle + s_Y(p) \le s_X(p) + \langle p, b_0\rangle. \tag{3.10}$$

Hence, $\langle p, z - b_0\rangle + s_Y(p) \le s_X(p)$. By the relationship between support functions and the Minkowski sum [11], we obtain

$$z - b_0 + Y \subset X.$$

Therefore, $z - b_0 \in X\dot{-}Y$. Consequently,

$$z = (z - b_0) + b_0 \in (X\dot{-}Y) + \overline{B}_r(0),$$

that is,

$$z \in (X\dot{-}Y) + \overline{B}_r(0).$$

Thus,

$$(X + \overline{B}_r(0))\dot{-}Y \subset (X\dot{-}Y) + \overline{B}_r(0).$$

Now consider a sequence of numbers θ_n satisfying

$$r > \theta_1 > \theta_2 > \cdots > \theta_n > \cdots, \qquad \lim_{n\to\infty} \theta_n = 0.$$

Clearly,

$$\overline{B}_{r-\theta_1}(0) \subset \overline{B}_{r-\theta_2}(0) \subset \cdots \subset \overline{B}_{r-\theta_n}(0) \subset \cdots,$$

and

$$\bigcup_n \overline{B}_{r-\theta_n}(0) = B_r(0).$$

For every natural number n,

$$(X + \overline{B}_{r-\theta_n}(0))\dot{-}Y \subset (X\dot{-}Y) + \overline{B}_{r-\theta_n}(0).$$

Moreover,

$$\bigcup_n \left(\left(X + \bar{B}_{r-\theta_n}(0)\right)\dot{-}Y\right) = \left(X + \bigcup_n \bar{B}_{r-\theta_n}(0)\right)\dot{-}Y = (X + B_r(0))\dot{-}Y, \tag{3.11}$$

and

$$\bigcup_n \left((X\dot{-}Y) + \overline{B}_{r-\theta_n}(0)\right) = (X\dot{-}Y) + \bigcup_n \overline{B}_{r-\theta_n}(0) = (X\dot{-}Y) + B_r(0). \tag{3.12}$$

Hence,

$$(X + B_r(0))\dot{-}Y \subset (X\dot{-}Y) + B_r(0).$$

The reverse inclusion follows from the property stated in Proposition 1.1.1 of reference [11]. Thus, Lemma 3.3 is proved. □

Remark 3.1. Equality (3.11) does not hold in the general case. However, in this situation it is valid because the balls $\overline{B}_{r-\theta_n}(0)$ are nested.

Example 3.2. Let $X = [0,1]$ and $Y = [0,2]$ be convex compact sets in the real line $\mathbb{R}$ and let $A_n = \left\{\left[2 - \frac{1}{n}, \frac{5}{2} - \frac{1}{n}\right], n \in \mathbb{N}\right\}$ be a family of closed intervals. The sets belonging to this family are not nested. For these sets, the relation $\bigcup_n \big((X + A_n) \dot{-} Y\big) \neq (X + \bigcup_n A_n) \dot{-} Y$ holds.

Indeed, let us compute the left-hand side of this relation. For each $n \in \mathbb{N}$,

$$X + A_n = \left[2 - \frac{1}{n}, \frac{7}{2} - \frac{1}{n}\right].$$

Hence, for every $n \in \mathbb{N}$, the Minkowski difference $(X + A_n) \dot{-} Y$ is empty. Therefore,

$$\bigcup_n \big((X + A_n) \dot{-} Y\big) = \varnothing.$$

Now consider the right-hand side. Since $\bigcup_n A_n = \left[1, \frac{5}{2}\right)$, we obtain

$$X + \bigcup_n A_n = [0,1] + \left[1, \frac{5}{2}\right) = \left[1, \frac{7}{2}\right).$$

Consequently,

$$\left(X + \bigcup_n A_n\right) \dot{-} Y = \left[1, \frac{7}{2}\right) \dot{-} [0,2] = \left[1, \frac{3}{2}\right].$$

Therefore, the equality does not hold in general when the sets A_n are not nested.

Now, we state the main achievement of the paper. Such type of results do not known. The continuity of the operation supports preserving many topological properties of spaces. This demonstrates the importance of the result obtained.

Theorem 3.2. *The Minkowski difference defined on the family $cc(\mathbb{R}^n)$ of compact convex sets is continuous.*

Proof. Let $X, Y \in cc(\mathbb{R}^n)$, and consider an arbitrary open neighborhood of the Minkowski difference $X \dot{-} Y \in cc(\mathbb{R}^n)$:

$$U = B_h(X \dot{-} Y, \varepsilon) = \{Z : h(Z, X \dot{-} Y) < \varepsilon\}.$$

Now consider the neighborhood of the pair $(X, Y) \in cc(\mathbb{R}^n) \times cc(\mathbb{R}^n)$:

$$V = B_h(X, \varepsilon/2) \times B_h(Y, \varepsilon/2).$$

Let us denote $\delta = \varepsilon/2$. For an arbitrary pair $(X', Y') \in V$, we have

$$h(X', X) < \delta, \qquad h(Y', Y) < \delta.$$

These relations can be written as

$$X' \subset X + B_\delta(0), \tag{3.13}$$

$$X \subset X' + B_\delta(0), \tag{3.14}$$

and

$$Y' \subset Y + B_\delta(0), \tag{3.15}$$

$$Y \subset Y' + B_\delta(0). \tag{3.16}$$

Assume that $z \in X' \dot{-} Y'$. Then, by the definition of the Minkowski difference, $z + Y' \subset X'$. Using relation (3.13), we obtain

$$z + Y' \subset X + B_\delta(0). \tag{3.17}$$

Adding $B_\delta(0)$ to both sides of (3.17) and using relation (3.16), we get

$$z + Y \subset X + B_{2\delta}(0). \tag{3.18}$$

Since the sets X and Y are compact and convex, by Lemma 3.1 relation (3.18) can be written as

$$z \in (X \dot{-} Y) + B_{2\delta}(0). \tag{3.19}$$

Because $\delta = \varepsilon/2$, we obtain

$$X' \dot{-} Y' \subset (X \dot{-} Y) + B_\varepsilon(0). \tag{3.20}$$

Similarly, one can derive

$$X \dot{-} Y \subset (X' \dot{-} Y') + B_\varepsilon(0). \tag{3.21}$$

Relations (3.20) and (3.21) imply that

$$h(X' \dot{-} Y', X \dot{-} Y) < \varepsilon.$$

The proof is complete. □

4. Conclusion

We show that, unlike the Minkowski sum, the Minkowski difference does not always preserve the subset density property. Consequently, we had to propose a condition under which the subset density property is preserved when using the Minkowski difference.

The fact that the Minkowski sum defined on the class of compact convex sets is a continuous mapping follows from the definition of a topological vector space and can be verified through several straightforward arguments. However, verifying that the Minkowski difference is a continuous mapping on the family $cc(\mathbb{R}^n)$ requires a number of more complicated steps. We establish appeared here problem which states that the Minkowski difference is continuous.

In addition, several geometric properties facilitating the computation of the Minkowski difference were presented in this work. These properties can be used not only for computing the Minkowski difference, but also in solving problems in functional analysis, convex analysis and optimal control theory.

Since the main result of the article is the continuity of the Minkowski difference, it becomes possible to apply the Minkowski difference in category theory.

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