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Research article

Pfaffian and Computational Analysis of Determinants of Skew-Symmetric Matrices for Homogeneous Tournaments

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Abstract

This paper investigates the properties of skew-symmetric matrices with a focus on the case when the order $m = 6$. After discussing the fundamental characteristics of skew-symmetric matrices, we derive the structure of their determinants for even values of m, particularly when $m = 2$ and 4, as preliminary cases to support our main study of the case $m = 6$. Furthermore, we introduce a tournament representation of such matrices, linking matrix entries to directed graphs based on their signs. We studied all non-isomorphic tournaments of order 6 and identified six of them as homogeneous, computed their corresponding matrices and determinants.

Keywords: *skew-symmetric matrix, determinant, Pfaffian, homogeneuos tournament, directed graph.*

AMS Subject Classification (2020): *Primary: 37B25 ; Secondary: 37C25; 37C27.*

1. Introduction

Pfaffian theory was first named after the German mathematician Johann Friedrich Pfaff (1765–1825), but its modern form and application in graph theory emerged later. AAccording to the literature, the theory of Pfaffian orientations was introduced by Pieter W. Kasteleyn (1924–1996) in the early 1960s [1]. Kasteleyn developed this theory to address enumeration problems in statistical physics, particularly related to the two-dimensional Ising model and dimer statistics [2].

Kasteleyn established foundational results for planar graphs and extended his approach to toroidal grids [1,2]. His work enabled the computation of perfect matching in graphs using Pfaffians, providing an efficient method to calculate determinants of skew-symmetric matrices. Skew-symmetric matrices play an essential role in various areas of mathematics and physics.

Before presenting the main results, let us start with preliminary information and a review of the literature.

A real value matrix A is called skew-symmetric if it satisfies $A = -A^T$, where A^T denotes the transpose of A. This implies that $a_{ii} = 0$ and $a_{ij} = -a_{ji}$ for all $i \neq j$ [4].

Such matrices are inherently square and have a structure that directly influences their determinant and applications in graph theory.

The goal of this paper is to analyze skew-symmetric matrices for small even values of m, with a primary focus on $m = 2n$. The cases $m = 2$, $m = 4$ and $m = 6$ are discussed briefly to provide foundational insight leading

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up to our main analysis. We also explore how their structure leads to a natural representation as tournaments (complete directed graphs).

We consider a general skew-symmetric matrix of order m:

$$A = [a_{ij}] \quad \text{such that} \quad a_{ij} = -a_{ji}, \quad a_{ii} = 0$$

From this condition, it follows that all the diagonal elements of the matrix must be zero and the matrix takes the following form:

$$\begin{bmatrix} 0 & a_{12} & a_{13} & \dots & a_{1m} \\ -a_{12} & 0 & a_{23} & \dots & a_{2m} \\ -a_{13} & -a_{23} & 0 & \dots & a_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{1m} & -a_{2m} & -a_{3m} & \dots & 0 \end{bmatrix}$$

The determinant of a skew-symmetric matrix of odd order is always equal to zero. Now, let m be even. In this case, the determinant of an m-order skew-symmetric matrix can be expressed as the square of a homogeneous polynomial of degree $m/2$ in its elements. The determinant of a skew-symmetric matrix of even order is expressed as the square of a polynomial known as the Pfaffian.

A graph $G = (V, E)$ is defined as a finite set V of vertices and a set E of edges, where each edge is an unordered pair $\{u, v\}$ of distinct vertices $u, v \in V$. Edges represent symmetric connections without direction, and in a simple graph, no loops or multiple edges between the same vertices are permitted [5].

Building on this, a directed graph (or digraph) $D = (V, A)$ introduces directionality, consisting of a finite vertex set V and a set A of arcs, where each arc is an ordered pair (u, v). An arc (u, v) indicates a directed connection from u to v. In a simple digraph, there are no loops (v, v) or multiple arcs in the same direction between the same vertices [6].

A tournament is a special type of directed graph, specifically a complete directed graph $T = (V, A)$, where for every pair of distinct vertices $\{u, v\} \subset V$, there exists exactly one arc, either $(u, v) \in A$ or $(v, u) \in A$. This ensures a unique directed relationship between every pair of vertices. For a tournament with n vertices, the number of arcs is $\binom{n}{2} = \frac{n(n-1)}{2}$ [3].

Let x_1, x_2 be the vertices of a tournament. The notation $x_1 \to x_2$ means that the edge connecting x_1 and x_2 is directed from x_1 to x_2. A finite sequence of vertices $x_1 \to x_2 \to \cdots \to x_p$ is called a *path* if $x_i \neq x_j$ for all $i \neq j$. A *cycle* is a closed path, i.e., $x_p = x_1$.

A tournament is called *strong* if, for any vertices $x, y \in Y$, there exists a path from x to y.

A tournament that contains no cycles is called *transitive*.

A tournament is called *homogeneous* if every sub-tournament is either strong or transitive.

Let $G = (V, E)$ be a graph, where V is the set of vertices and E is the set of edges. A *Hamiltonian cycle* in G is a cycle that:

$$v_1 \to v_2 \to \cdots \to v_n \to v_1$$

where:

- $\{v_1, v_2, \dots, v_n\} = V$, meaning all vertices in the graph are visited exactly once.
- For each $i \in \{1, 2, \dots, n-1\}$, the edges $(v_i, v_{i+1}) \in E$.
- The edge $(v_n, v_1) \in E$, completing the cycle.

Two tournaments are isomorphic if there is a relabelling of the vertices that preserves the directed edges. That is, if there exists a bijection (one-to-one mapping) $f : V_1 \to V_2$ between the vertex sets of two tournaments T_1 and T_2 such that:

$$x \to y \text{ in } T_1 \quad \Longleftrightarrow \quad f(x) \to f(y) \text{ in } T_2.$$

If no such mapping exists, the tournaments are non-isomorphic. The number of non-isomorphic tournaments grows as m increases.The following table in Moon [3] shows the number of all non-isomorph tournaments:

m	Non-isomorphic tournaments
2	1
3	2
4	4
5	12
6	56

From the table, it can be seen that when m=6, the number of non-isomorphic tournaments is 56. Out of these 56 tournaments, we took 6 homogeneous ones; we find the determinants of their corresponding skew-symmetric matrices using the Pfaffian.

2. Pfaffian

Definition 2.1. [1] For a $2n \times 2n$ skew-symmetric matrix A (that is, $A^T = -A$), the Pfaffian $\mathrm{Pf}(A)$ is a polynomial in the matrix entries such that its square is equal to the determinant: $\mathrm{Pf}(A)^2 = \det(A)$.

For an odd-dimensional skew-symmetric matrix ($m \times m$, m odd), the Pfaffian is defined as zero because $\det(A) = 0$.

Explicitly, for a $2n \times 2n$ matrix $A = (a_{ij})$, the Pfaffian is:

$$\mathrm{Pf}(A) = \frac{1}{2^n n!} \sum_{\sigma \in S_{2n}} \mathrm{sgn}(\sigma) \prod_{i=1}^{n} a_{\sigma(2i-1),\sigma(2i)},$$

where S_{2n} is the symmetric group, and $\mathrm{sgn}(\sigma)$ is the signature of permutation σ. Alternatively, it can be expressed on partitions of $\{1, \ldots, 2n\}$ into pairs. For a matrix 0×0, $\mathrm{Pf}(A) = 1$ is the convention.

The expansion of $\mathrm{Pf}(A)^2$ results in 56 monomials involving six variables each. Listing all these terms explicitly is feasible but lengthy; here we provide symbolic representation:

$$\det(A) = \sum_{i=1}^{56} c_i \cdot a_{i_1 i_2} a_{i_3 i_4} a_{i_5 i_6} a_{j_1 j_2} a_{j_3 j_4} a_{j_5 j_6}.$$

Examples:

For a 2×2 skew-symmetric matrix $A = \begin{bmatrix} 0 & a_{12} \\ -a_{12} & 0 \end{bmatrix}$, $\mathrm{Pf}(A) = a_{12}$.

For a 4×4 skew-symmetric matrix $A = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} \\ -a_{12} & 0 & a_{23} & a_{24} \\ -a_{13} & -a_{23} & 0 & a_{34} \\ -a_{14} & -a_{24} & -a_{34} & 0 \end{bmatrix}$, $\mathrm{Pf}(A) = a_{12}a_{34} - a_{13}a_{24} + a_{14}a_{23}$.

The determinant of a 6 × 6 skew-symmetric matrix consists of the square of the following Pfaffian expression:

$$A = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ -a_{12} & 0 & a_{23} & a_{24} & a_{25} & a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & a_{36} \\ -a_{14} & -a_{24} & -a_{34} & 0 & a_{45} & a_{46} \\ -a_{15} & -a_{25} & -a_{35} & -a_{45} & 0 & a_{56} \\ -a_{16} & -a_{26} & -a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$

$$\begin{aligned} \mathrm{Pf}(A) = {} & a_{12}a_{34}a_{56} - a_{12}a_{35}a_{46} + a_{12}a_{36}a_{45} + a_{13}a_{24}a_{56} \\ & - a_{13}a_{25}a_{46} + a_{13}a_{26}a_{45} + a_{14}a_{23}a_{56} - a_{14}a_{25}a_{36} \\ & + a_{14}a_{26}a_{35} + a_{15}a_{23}a_{46} - a_{15}a_{24}a_{36} + a_{15}a_{26}a_{34} \\ & + a_{16}a_{23}a_{45} - a_{16}a_{24}a_{35} + a_{16}a_{25}a_{34}. \end{aligned}$$

Hence, the determinant of the skew-symmetric matrix is equal to the square of the Pfaffian:

$$\det(A) = \mathrm{Pf}(A)^2.$$

3. Main results

3.1. Python script for tournament graph generation and visualization

This Python script automates the creation and visualization of tournament graphs, which are complete directed graphs used in combinatorics and game theory. The script allows users to input a skew-symmetric adjacency matrix (where $a_{ij} = -a_{ji}$), ensuring the graph represents a valid tournament. Using networkx and matplotlib, the program generates an interactive visualization of the graph, displaying nodes in a circular layout with directed edges. Additionally, the script produces LaTeX Tikz code, enabling seamless integration into academic papers or presentations. By converting the adjacency matrix into a structured TikZ diagram, researchers can efficiently document and analyze tournament structures without manual drawing. This tool is particularly useful for mathematicians and computer scientists studying graph theory, game tournaments, or discrete dynamical systems, as it bridges computational analysis with formal typesetting. The generated LaTeX output can be directly compiled to produce publication-ready figures, streamlining the workflow for theoretical research.

Key Features:

User-Friendly Input: The script prompts for the upper triangular part of the adjacency matrix, automatically enforcing skew-symmetry.

Visualization: Use networkx to render the graph, aiding in immediate visual verification.

LaTeX Integration: Output TikZ code for high-quality vector graphics in academic documents.

Reproducibility: Saves the LaTeX code to a file, ensuring that the results are reproducible and editable.

This approach exemplifies how scripting can enhance mathematical research by combining computational tools with traditional publishing formats. Future extensions could include automated analysis of graph properties (e.g., Hamiltonian cycles) or support for weighted edges.

The program consists of the following steps:

1. Construction of a Skew-Symmetric Matrix.

The skew-symmetric matrix satisfies the following condition:

$$A^T = -A$$

This means that if $a_{ij} = 1$, then $a_{ji} = -1$. The user only inputs the upper triangular values, and the rest are automatically filled:

```
def get_adjacency_matrix(n):
    A = np.zeros((n, n), dtype=int)
    for i in range(n):
        row = list(map(int, input(f"Row {i+1} (only {n-i-1} values): ").split()))
        for j in range(i + 1, n):
            A[i, j] = row[j - (i + 1)]
            A[j, i] = -A[i, j]
    return A
```

2. Drawing the Tournament Graph.
The following code uses `networkx` and `matplotlib` to visualize the tournament graph:

```
def draw_tournament_graph(A):
    G = nx.DiGraph()
    for i in range(len(A)):
        for j in range(len(A)):
            if A[i, j] == 1:
                G.add_edge(i, j)
    pos = nx.circular_layout(G)
    nx.draw(G, pos, with_labels=True, node_color='lightblue', edge_color='gray',
    arrows=True)
    plt.show()
```

3.LaTeX TikZ Code Generation.
To use the tournament graph in academic papers we need to draw it in format, because of this we generate TikZ code of it:

```
def adjacency_to_latex(A):
    latex_code = "\\begin{center}\\textbf{T1}\n"
    latex_code += "\\begin{tikzpicture}[->,>=stealth',shorten >=1pt,auto,
    node distance=1cm,\n"
    latex_code += "thick,main node/.style={circle,draw,minimum size=0.5cm,
    font=\\sffamily\\scriptsize}]\n\n"
    angles = np.linspace(0, 360, len(A), endpoint=False)
    for i in range(len(A)):
        latex_code += f"\\node[main node] ({i}) at ({angles[i]}:1.5cm)
        {{{i+1}}};\n"
    for i in range(len(A)):
        for j in range(len(A)):
            if A[i, j] == 1:
                latex_code += f"\\path ({i}) edge ({j});\n"
    latex_code += "\\end{tikzpicture}\\end{center}"
    return latex_code
```

4.Main function.
The main function integrates all components. It receives input data and outputs the final result.

```
def main():
    n = int(input("Enter the number of nodes: "))
    A = get_adjacency_matrix(n)
    print("Adjacency Matrix:")
    print(A)
    draw_tournament_graph(A)
    latex_code = adjacency_to_latex(A)
    print("\nLaTeX Code:")
    print(latex_code)
    with open("tournament_graph.tex", "w") as f:
        f.write(latex_code)
    print("\nLaTeX code saved to tournament_graph.tex")
```

After program run:

- User inputs number of nodes;
- Skew-symmetric matrix is generated;
- Graph corresponding the matrix is drawn;
- LaTeX TikZ code is generated and saved into file.

Example execution: If $n = 5$ and with input:

```
Row 1 (only 4 values): 1 1 -1 1
Row 2 (only 3 values): -1 -1 1
Row 3 (only 2 values): 1 1
Row 4 (only 1 values): -1
Row 5 (only 0 values):
```

The resulting matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 & -1 & 1 \\ -1 & 0 & -1 & -1 & 1 \\ -1 & 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 1 & 0 \end{bmatrix}$$

The graph of this skew-symmetric matrix is shown in Figure 1.

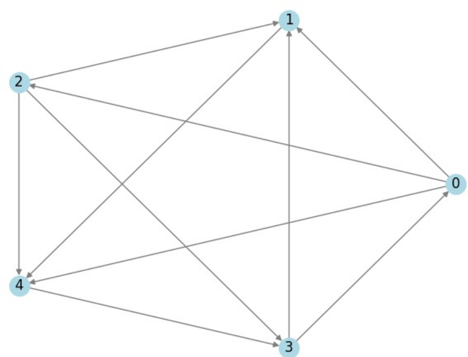


Figure 1. Tournament graph generated from the skew-symmetric matrix using Python.

The program produces a ready-to-use TikZ code:

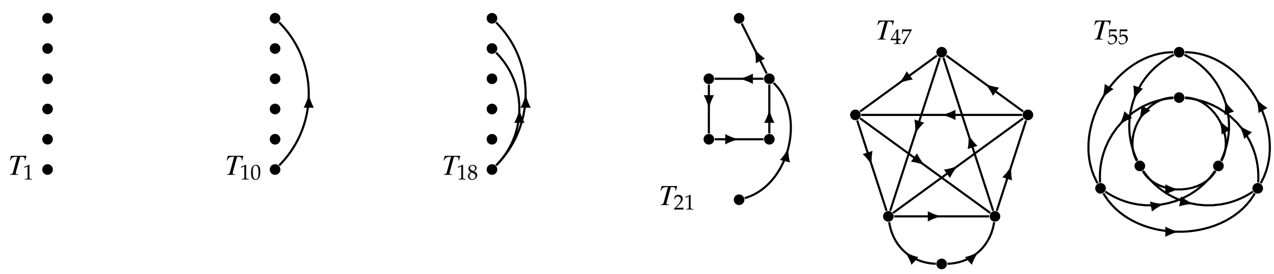
```
LaTeX Code:
\begin{center}\textbf{T}
\begin{tikzpicture}[->,>=stealth',shorten >=1pt,auto,node distance=1cm,
thick,main node/.style={circle,draw,minimum size=0.5cm, font=\sffamily\scriptsize

\node[main node] (0) at (0.0:1.5cm) {1};
\node[main node] (1) at (72.0:1.5cm) {2};
\node[main node] (2) at (144.0:1.5cm) {3};
\node[main node] (3) at (216.0:1.5cm) {4};
\node[main node] (4) at (288.0:1.5cm) {5};
\path (0) edge (1);
\path (0) edge (2);
\path (0) edge (4);
\path (1) edge (4);
\path (2) edge (1);
\path (2) edge (3);
\path (2) edge (4);
\path (3) edge (0);
\path (3) edge (1);
\path (4) edge (3);
\end{tikzpicture}\end{center}
```

This Python program enables the export of a tournament graph for use in LaTeX. The generated program allows the user to create visually appealing diagrams in a LaTeX environment (e.g., Overleaf or other LaTeX compilers).

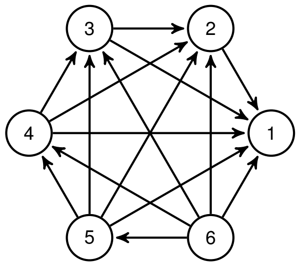
3.2. Homogeneous Tournaments

In J. Moon's monograph [3], fifty-six non-isomorphic tournament graphs with six vertices are presented, each possessing distinct properties. In this article, we have used these tournament graphs to identify and separate homogeneous tournaments among them. The following drawing drawings are used to illustrate tournaments. Not all of the arcs have been included in the drawings; if an arc joining two nodes has not been drawn, then it should be understood that the arc is oriented from the higher node to the lower node.



To study the dynamics of homogeneous tournaments, we visualized the graphs in a convenient form using Python software. Additionally, we calculated the determinants of the corresponding skew-symmetric matrices for these homogeneous tournaments using the Pfaffian method.

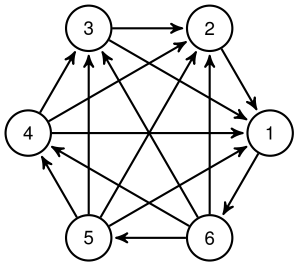
$$T_1 = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ -a_{12} & 0 & a_{23} & a_{24} & a_{25} & a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & a_{36} \\ -a_{14} & -a_{24} & -a_{34} & 0 & a_{45} & a_{46} \\ -a_{15} & -a_{25} & -a_{35} & -a_{45} & 0 & a_{56} \\ -a_{16} & -a_{26} & -a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$



T1

$det(T_1) = (a_{16}a_{25}a_{34} - a_{15}a_{26}a_{34} + a_{12}a_{56}a_{34} - a_{16}a_{24}a_{35} + a_{14}a_{26}a_{35} + a_{15}a_{24}a_{36} - a_{14}a_{25}a_{36} + a_{16}a_{23}a_{45} - a_{13}a_{26}a_{45} + a_{12}a_{36}a_{45} - a_{15}a_{23}a_{46} + a_{13}a_{25}a_{46} - a_{12}a_{35}a_{46} + a_{14}a_{23}a_{56} - a_{13}a_{24}a_{56})^2.$

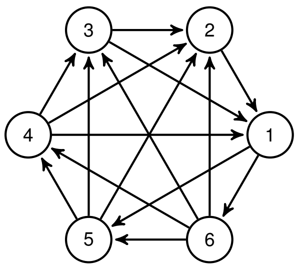
$$T_{10} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & a_{15} & -a_{16} \\ -a_{12} & 0 & a_{23} & a_{24} & a_{25} & a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & a_{36} \\ -a_{14} & -a_{24} & -a_{34} & 0 & a_{45} & a_{46} \\ -a_{15} & -a_{25} & -a_{35} & -a_{45} & 0 & a_{56} \\ a_{16} & -a_{26} & -a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$



T10

$det(T_{10}) = (-a_{14}a_{26}a_{35} + a_{12}a_{46}a_{35} + a_{14}a_{25}a_{36} + a_{13}a_{26}a_{45} - a_{12}a_{36}a_{45} + a_{16}(a_{25}a_{34} - a_{24}a_{35} + a_{23}a_{45}) - a_{13}a_{25}a_{46} + a_{15}(a_{26}a_{34} - a_{24}a_{36} + a_{23}a_{46}) - a_{14}a_{23}a_{56} + a_{13}a_{24}a_{56} - a_{12}a_{34}a_{56})^2.$

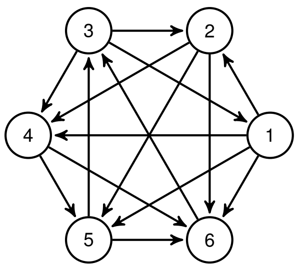
$$T_{18} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & -a_{15} & -a_{16} \\ -a_{12} & 0 & a_{23} & a_{24} & a_{25} & a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & a_{36} \\ -a_{14} & -a_{24} & -a_{34} & 0 & a_{45} & a_{46} \\ a_{15} & -a_{25} & -a_{35} & -a_{45} & 0 & a_{56} \\ a_{16} & -a_{26} & -a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$



T18

$det(T_{18}) = (-a_{16}a_{25}a_{34} + a_{15}a_{26}a_{34} + a_{12}a_{56}a_{34} + a_{16}a_{24}a_{35} + a_{14}a_{26}a_{35} - a_{15}a_{24}a_{36} - a_{14}a_{25}a_{36} - a_{16}a_{23}a_{45} - a_{13}a_{26}a_{45} + a_{12}a_{36}a_{45} + a_{15}a_{23}a_{46} + a_{13}a_{25}a_{46} - a_{12}a_{35}a_{46} + a_{14}a_{23}a_{56} - a_{13}a_{24}a_{56})^2.$

$$T_{21} = \begin{bmatrix} 0 & -a_{12} & a_{13} & -a_{14} & -a_{15} & -a_{16} \\ a_{12} & 0 & a_{23} & -a_{24} & -a_{25} & -a_{26} \\ -a_{13} & -a_{23} & 0 & -a_{34} & a_{35} & a_{36} \\ a_{14} & a_{24} & a_{34} & 0 & -a_{45} & -a_{46} \\ a_{15} & a_{25} & -a_{35} & a_{45} & 0 & -a_{56} \\ a_{16} & a_{26} & -a_{36} & a_{46} & a_{56} & 0 \end{bmatrix},$$

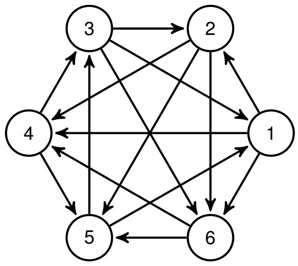


T21

$det(T_{21}) = (-a_{16}a_{25}a_{34} + a_{15}a_{26}a_{34} - a_{12}a_{56}a_{34} - a_{16}a_{24}a_{35} + a_{14}a_{26}a_{35} + a_{15}a_{24}a_{36} + a_{14}a_{25}a_{36} + a_{16}a_{23}a_{45} - a_{13}a_{26}a_{45} + a_{12}a_{36}a_{45} - a_{15}a_{23}a_{46} + a_{13}a_{25}a_{46} + a_{12}a_{35}a_{46} + a_{14}a_{23}a_{56} - a_{13}a_{24}a_{56})^2.$

$$T_{47} = \begin{bmatrix} 0 & -a_{12} & a_{13} & -a_{14} & a_{15} & -a_{16} \\ a_{12} & 0 & a_{23} & -a_{24} & -a_{25} & -a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & -a_{36} \\ a_{14} & a_{24} & -a_{34} & 0 & -a_{45} & a_{46} \\ -a_{15} & a_{25} & -a_{35} & a_{45} & 0 & a_{56} \\ a_{16} & a_{26} & a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$

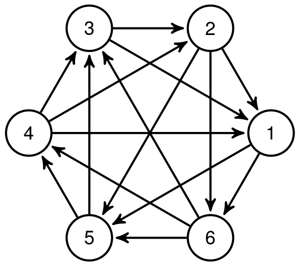
T47



$det(T_{47}) = (a_{14}a_{26}a_{35} + a_{12}a_{46}a_{35} - a_{14}a_{25}a_{36} + a_{13}a_{26}a_{45} + a_{12}a_{36}a_{45} - a_{16}a_{25}a_{34} - a_{16}a_{24}a_{35} - a_{16}a_{23}a_{45} + a_{13}a_{25}a_{46} - a_{15}a_{26}a_{34} - a_{15}a_{24}a_{36} + a_{15}a_{23}a_{46} + a_{14}a_{23}a_{56} - a_{13}a_{24}a_{56} + a_{12}a_{34}a_{56})^2.$

$$T_{55} = \begin{bmatrix} 0 & a_{12} & a_{13} & a_{14} & -a_{15} & -a_{16} \\ -a_{12} & 0 & a_{23} & a_{24} & -a_{25} & -a_{26} \\ -a_{13} & -a_{23} & 0 & a_{34} & a_{35} & a_{36} \\ -a_{14} & -a_{24} & -a_{34} & 0 & a_{45} & a_{46} \\ a_{15} & a_{25} & -a_{35} & -a_{45} & 0 & a_{56} \\ a_{16} & a_{26} & -a_{36} & -a_{46} & -a_{56} & 0 \end{bmatrix},$$

T55



$det(T_{55}) = (a_{16}a_{25}a_{34} - a_{15}a_{26}a_{34} + a_{12}a_{56}a_{34} + a_{16}a_{24}a_{35} - a_{14}a_{26}a_{35} - a_{15}a_{24}a_{36} + a_{14}a_{25}a_{36} - a_{16}a_{23}a_{45} + a_{13}a_{26}a_{45} + a_{12}a_{36}a_{45} + a_{15}a_{23}a_{46} - a_{13}a_{25}a_{46} - a_{12}a_{35}a_{46} + a_{14}a_{23}a_{56} - a_{13}a_{24}a_{56})^2.$

4. Conclusion

In this paper, we conducted a comprehensive analysis of the determinants of skew-symmetric matrices associated with homogeneous tournaments, utilizing the Pfaffian method for computation. Using Python scripting with libraries such as NetworkX and Matplotlib, we successfully visualized homogeneous tournament graphs with the help of the NetworkX library and enabled their visualization using the Matplotlib library. The automated generation of LaTeX TikZ code further facilitated the production of publication-ready diagrams, enhancing the efficiency of documenting complex tournament structures. In our future work, we will continue to expand these methods utilizing the results obtained above to model the dynamics of strongly homogeneous tournaments. Using computational tools such as Python scripts, we plan to implement matrix input through a graphical user interface (GUI), identify tournament properties (e.g., Hamiltonian cycles), and automatically generate random tournament graphs.

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References

[1] Kasteleyn, P. W. *Graph Theory and Crystal Physics.* In Graph Theory and Theoretical Physics. (1967).
[2] Kasteleyn, P. W. *Dimer Statistics and Phase Transitions*. Journal of Mathematical Physics. **4(2)**, p.287–293,(1963).
[3] Moon,J. W. *Topics on Tournaments*, New York: Holt, Rinehart and Winston, p. 112. (1968).
[4] Horn, R. A., & Johnson, C. R. *Matrix Analysis*. Cambridge University. (2012).
[5] Harary F., Palmer E.M. *Graphical enumeration*. Academic Press New York and London. (1973).
[6] G. Chartrand and H. Jordon and V. Vatter and P. Zhang. *Graphs and Digraphs*. CRC Press. p. 364. (2024).
[7] Koh Kh., Dong F., and Tay E.G. *Introduction to graph theory*. World Scientific. p. 308. (2024).
[8] Ganikhodzhaev R.N. *Quadratic stochastic operators, Lyapunov function and tournaments*, Acad. Sci. Sb. Math., **76(2)**, p. 489-506. (1993)
[9] Ganikhodzhaev R.N., Tadzhieva M.A. *Stability of fixed points of discrete dynamic systems of Volterra type*. AIP Conference Proceedings, 2021. V. **2365**. P. 060005-1, 060005-7. https://doi.org/10.1063/5.0057979. .
[10] Ganikhodzhaev R.N., Tadzieva M.A., Eshmamatova D.B. *Dynamical Proporties of Quadratic Homeomorphisms of a Finite-Dimensional Simplex*. Journal of Mathematical Sciences, **245(3)**. P. 398-402.

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