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Research article

A Nonlocal Problem for a Second-Order Nonlinear Ordinary Differential Equation with Degeneration at the Boundary of the Domain

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Abstract

This paper investigates a nonlocal boundary value problem for a nonlinear second-order ordinary differential equation with degenerating coefficients at the boundaries of the interval. The differential equation is defined on the interval $(0, 1)$ with degeneracy at both endpoints $x = 0$ and $x = 1$. The problem is reduced to an integral equation using the Green's function method. Under appropriate conditions on the nonlinear term, the existence and uniqueness of the solution are established based on the contraction mapping principle. Explicit estimates for the solution are provided using Gronwall's inequality.

Keywords: *nonlinear differential equation; degenerating coefficients; nonlocal boundary conditions; Green's function; integral equation; contraction mapping.*

AMS Subject Classification (2020): *Primary: 34B10 ; Secondary: 34B27; 34B15; 45G10; 47H10.*

1. Introduction

Boundary value problems for differential equations with degenerating coefficients occupy an important place in modern mathematical analysis, since they connect fundamental theory with a broad range of applications. In classical models the differential operator is assumed to be non-degenerate, so that the order of the equation is preserved throughout the domain. In many problems of mathematical physics, mechanics, and biology, however, the coefficients of the highest-order derivatives vanish on the boundary of the domain, which causes a loss of order and gives rise to singular behaviour requiring specialized analytical tools. Such degenerate models arise, for instance, in filtration theory and heat-conduction processes, in the bending theory of thin shells and elastic plates of variable thickness, and in the description of transport processes whose diffusion rate tends to zero at the spatial boundaries [1], [2].

Classical boundary value problems are usually posed with local boundary conditions. In many applied situations such conditions are not sufficient, and one is led to nonlocal conditions formulated through integral, functional, or coupled boundary relations, which make it possible to take into account memory effects and global properties of complex processes [3, 4, 5]. Nonlocal problems for degenerate and mixed-type equations, including those with integral conditions and nonlinear kernels, have been actively studied in recent years [6, 7, 8, 9, 10].

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A central role in the study of such problems is played by the Green's function method, which reduces a boundary value problem to an equivalent integral equation and thereby makes the tools of functional analysis available . When the weight function is singular, the very construction of the Green's function becomes one of the main difficulties . For nonlinear equations, the existence and uniqueness of a solution are then established by fixed-point techniques, most often the Banach contraction mapping principle. Related questions for second-order functional and singular equations have also been considered [11, 12].

Most existing works deal with equations possessing a single line of degeneracy or with linear cases. Significant progress in the study of degenerate equations of higher even order, as well as of equations with a single line of degeneracy, has been achieved recently [13]. Problems combining two-sided degeneration, nonlinearity, and nonlocal boundary conditions have been studied considerably less. Models with two lines of degeneracy, characterized by the weight $x^\alpha(1-x)^\beta$ vanishing at both endpoints $x=0$ and $x=1$, were considered for higher-order and fourth-order equations in [14, 15].

In the present paper we consider a nonlocal boundary value problem for a nonlinear second-order ordinary differential equation defined on the interval $(0,1)$, with degeneracy at both endpoints $x=0$ and $x=1$.

2. Statement of the problem

Consider the following nonlinear second-order ordinary differential equation with degeneration at both endpoints of the interval $(0,1)$:

$$\left[x^\alpha(1-x)^\beta y'(x)\right]' = f(x,y(x)), \qquad 0<x<1, \tag{2.1}$$

where $y(\cdot)$ is an unknown function, $f(\cdot,\cdot)$ is a given function, α and β are given real numbers such that $0<\alpha<1$, $0<\beta<1$.

Problem 1 (Problem A_{pq})**.** *Find a function y belonging to the class $y,\ x^\alpha(1-x)^\beta y' \in C[0,1]$, $[x^\alpha(1-x)^\beta y']' \in C(0,1)$, satisfying equation (2.1) and the boundary conditions*

$$p\,y(0) = q\,y(1) + A, \tag{2.2}$$

$$q\lim_{x\to 0}[x^\alpha y'(x)] = p\lim_{x\to 1}[(1-x)^\beta y'(x)] + B, \tag{2.3}$$

where A, B, p, and q are given real numbers, and $p \neq q$.

3. Construction of the Green's function

First, let $f(x,y(x)) = F(x)$ and seek the function $y(x)$ in the form

$$y(x) = y_0(x) + y_F(x),$$

where $y_0(x)$ is a solution of the problem

$$\begin{cases} \left[x^\alpha(1-x)^\beta y_0'(x)\right]' = 0, \\ p\,y_0(0) = q\,y_0(1) + A, \quad q\lim\limits_{x\to 0}[x^\alpha y_0'(x)] = p\lim\limits_{x\to 1}[(1-x)^\beta y_0'(x)] + B, \end{cases} \tag{3.1}$$

and $y_F(x)$ is a solution of the problem

$$\begin{cases} \left[x^\alpha(1-x)^\beta y_F'(x)\right]' = F(x), \\ p\,y_F(0) = q\,y_F(1), \quad q\lim\limits_{x\to 0}[x^\alpha y_F'(x)] = p\lim\limits_{x\to 1}[(1-x)^\beta y_F'(x)], \end{cases} \tag{3.2}$$

It is not difficult to show that the general solution of the equation in problem (3.1) is given by

$$y_0(x) = C_2 + C_1 \int_0^x z^{-\alpha}(1-z)^{-\beta} dz. \tag{3.3}$$

Substituting this general solution into the boundary conditions in (3.1), we find C_1 and C_2:

$$C_1 = -\frac{B}{p-q}, \quad C_2 = \frac{A}{p-q} - \frac{Bq\sigma}{(p-q)^2},$$

where $\sigma = [\Gamma(1-\alpha)\Gamma(1-\beta)/\Gamma(2-\alpha-\beta)]$, and $\Gamma(z)$ is Euler's gamma function.

Substituting the obtained C_1 and C_2 into (3.3), we find the function $y_0(x)$ in the form:

$$y_0(x) = \frac{A}{p-q} - \frac{Bq\sigma}{(p-q)^2} - \frac{B}{p-q} \int_0^x z^{-\alpha}(1-z)^{-\beta} dz. \tag{3.4}$$

Now we proceed to solve problem (3.2) using the Green's function method.

Definition 3.1. The Green's function for problem (3.2) is defined as a function $G(x,s)$ satisfying the following conditions:

1°) $G(x,s)$ is continuous for $x, s \in [0,1]$;

2°) For each interval $(0,s)$ and $(s,1)$, the derivative $(\partial/\partial x)G(x,s)$ is continuous, and at $x = s$ there is a jump equal to $\left[s^{-\alpha}(1-s)^{-\beta}\right]$, i.e.,

$$\left.\frac{\partial G(x,s)}{\partial x}\right|_{x=s+0} - \left.\frac{\partial G(x,s)}{\partial x}\right|_{x=s-0} = s^{-\alpha}(1-s)^{-\beta}; \tag{3.5}$$

3°) On each interval $(0,s)$ and $(s,1)$, the function $G(x,s)$ satisfies the homogeneous equation with respect to x:

$$\left[x^{\alpha}(1-x)^{\beta} G_x(x,s)\right]_x = 0;$$

4°) For all $s \in (0,1)$,

$$pG(0,s) = qG(1,s), \quad q \lim_{x\to 0} \left[x^{\alpha} G_x(x,s)\right] = p \lim_{x\to 1} \left[(1-x)^{\beta} G_x(x,s)\right] \tag{3.6}$$

are satisfied.

We first derive the Green's function from first principles. Set

$$\Phi(x) = \int_0^x z^{-\alpha}(1-z)^{-\beta}\, dz, \qquad \Phi'(x) = x^{-\alpha}(1-x)^{-\beta}, \qquad \Phi(0) = 0, \;\; \Phi(1) = \sigma.$$

By condition 3°, on each of the subintervals $(0,s)$ and $(s,1)$ the function $G(\cdot,s)$ is a solution of $\left[x^{\alpha}(1-x)^{\beta} G_x\right]_x = 0$, whose general solution is a linear combination of 1 and $\Phi(x)$. Hence

$$G(x,s) = \begin{cases} a_1 + b_1\, \Phi(x), & 0 \le x < s, \\ a_2 + b_2\, \Phi(x), & s < x \le 1, \end{cases}$$

with coefficients a_1, b_1, a_2, b_2 depending on s. Since $\lim_{x\to 0}[x^{\alpha}\Phi'(x)] = \lim_{x\to 1}[(1-x)^{\beta}\Phi'(x)] = 1$, the second relation in 4° gives $q\, b_1 = p\, b_2$, while the jump condition 2° yields $b_2 - b_1 = 1$. Solving these two equations,

$$b_1 = -\frac{p}{p-q}, \qquad b_2 = -\frac{q}{p-q}.$$

The continuity condition 1° at $x = s$ gives $a_1 - a_2 = (b_2 - b_1)\Phi(s) = \Phi(s)$, and the first relation in 4°, namely $p\,G(0, s) = q\,G(1, s)$ with $G(0, s) = a_1$ and $G(1, s) = a_2 + b_2\sigma$, determines

$$a_2 = -\frac{p}{p-q}\,\Phi(s) - \frac{q^2\sigma}{(p-q)^2}, \qquad a_1 = -\frac{q}{p-q}\,\Phi(s) - \frac{q^2\sigma}{(p-q)^2}.$$

Substituting these coefficients yields formula (3.7) below.

Lemma 3.1. *The Green's function for problem* (3.2) *exists and is unique; it is defined by the formula:*

$$G(x,s) = \begin{cases} -\dfrac{p}{p-q}\displaystyle\int_0^x z^{-\alpha}(1-z)^{-\beta}dz - \dfrac{q}{p-q}\displaystyle\int_0^s z^{-\alpha}(1-z)^{-\beta}dz - \dfrac{q^2\sigma}{(p-q)^2}, & x < s; \\ -\dfrac{p}{p-q}\displaystyle\int_0^s z^{-\alpha}(1-z)^{-\beta}dz - \dfrac{q}{p-q}\displaystyle\int_0^x z^{-\alpha}(1-z)^{-\beta}dz - \dfrac{q^2\sigma}{(p-q)^2}, & s < x. \end{cases} \tag{3.7}$$

Proof. Since $\alpha, \beta \in [0, 1)$, each integral in formula (3.7) is a continuous function with respect to its upper limit. Therefore, the function $G(x, s)$ is continuous in the domain $\Omega_1 = \{(x, s) \colon 0 \le x \le 1,\ 0 \le s \le 1\}$ for $x \ne s$.

Furthermore, for each fixed $s \in (0, 1)$,

$$\lim_{x\to s+0} G(x,s) = \lim_{x\to s-0} G(x,s) = G(s,s)$$

holds, where

$$G(s,s) = -\frac{p+q}{p-q}\int_0^s z^{-\alpha}(1-z)^{-\beta}dz - \frac{q^2\sigma}{(p-q)^2}.$$

Since $G(s, s)$ is continuous for $s \in [0, 1]$, it follows that $G(x, s) \in C\big(\overline{\Omega}_1\big)$.

Now, by directly differentiating formula (3.7), we obtain:

$$\frac{\partial}{\partial x}G(x,s) = \begin{cases} -\dfrac{p}{p-q}\,x^{-\alpha}(1-x)^{-\beta}, & x < s, \\ -\dfrac{q}{p-q}\,x^{-\alpha}(1-x)^{-\beta}, & s < x. \end{cases} \tag{3.8}$$

From here, the validity of equality (3.5) follows easily.

According to (3.8),

$$\frac{\partial}{\partial x}\left[x^{\alpha}(1-x)^{\beta}\frac{\partial}{\partial x}G(x,s)\right] = 0,$$

holds for $x \in (0, s) \cup (s, 1)$, i.e., the function $G(x, s)$ satisfies the homogeneous equation with respect to x:

$$\frac{\partial}{\partial x}\left[x^{\alpha}(1-x)^{\beta}\frac{\partial}{\partial x}G(x,s)\right] = 0.$$

Using formulas (3.7) and (3.8), and also taking into account the equality

$$\int_0^1 z^{-\alpha}(1-z)^{-\beta}\,dz = \sigma,$$

we derive the following:

$$G(0,s) = -\frac{q}{p-q}\int_0^s \frac{dz}{z^\alpha(1-z)^\beta} - \frac{q^2\sigma}{(p-q)^2},$$

$$G(1,s) = -\frac{p}{p-q}\int_0^s \frac{dz}{z^\alpha(1-z)^\beta} - \frac{q\sigma}{p-q} - \frac{q^2\sigma}{(p-q)^2},$$

$$\lim_{x\to 0}\,[x^\alpha G_x(x,s)] = -\frac{p}{p-q}, \qquad \lim_{x\to 1}\left[(1-x)^\beta G_x(x,s)\right] = -\frac{q}{p-q}.$$

A direct check shows that with these values $p\,G(0,s) = q\,G(1,s)$ and $q\lim_{x\to 0}[x^\alpha G_x] = p\lim_{x\to 1}[(1-x)^\beta G_x]$, so that conditions (3.6) are fulfilled. Uniqueness follows from the fact that the four conditions 1°–4° determine the coefficients a_1, b_1, a_2, b_2 uniquely. □

Using the Green's function, we find the solution of problem (3.2) in the form:

$$y_F(x) = \int_0^1 G(x,s)F(s)ds.$$

Indeed, by (3.8),

$$x^\alpha(1-x)^\beta y_F'(x) = -\frac{q}{p-q}\int_0^x F(s)\,ds - \frac{p}{p-q}\int_x^1 F(s)\,ds,$$

whence $\left[x^\alpha(1-x)^\beta y_F'\right]' = \dfrac{-q+p}{p-q}F(x) = F(x)$, and the homogeneous nonlocal conditions in (3.2) hold because G satisfies (3.6).

Utilizing these solutions and the relations $y(x) = y_0(x) + y_F(x)$, $f(x, y(x)) = F(x)$, we reduce problem (2.1), (2.2) to the following equivalent integral equation:

$$y(x) = y_0(x) + \int_0^1 G(x,s)f(s,y(s))ds. \tag{3.9}$$

4. Existence and uniqueness of the solution

Now we examine the existence and uniqueness of the solution to integral equation (3.9). For this purpose, consider the following operator:

$$Ty = y_0(x) + \int_0^1 G(x,s)f(s,y(s))ds.$$

Definition 4.1 (Contraction mapping)**.** [16] Let X be a metric space and A a mapping from X into itself. If there exists a number $\delta \in (0,1)$ such that for all $y_1, y_2 \in X$ the inequality

$$\rho(Ay_1, Ay_2) \le \delta\rho(y_1, y_2)$$

holds, then A is called a contraction mapping, where $\rho(y_1, y_2)$ is the metric of the space X.

If for a mapping $A : X \to X$ there exists a point $x \in X$ such that $Ax = x$, then the point x is called a fixed point of the mapping A.

Lemma 4.1 (Contraction Mapping Principle). *[16] Every contraction mapping in a complete metric space has a unique fixed point.*

Lemma 4.2. *[16] The space $C[a,b]$ is complete.*

It is known that in the space $C[0,1]$, the metric $\rho(y_1, y_2)$ is defined by

$$\rho(y_1, y_2) = \max_{0\le x\le 1} |y_1 - y_2| .$$

We will show that the operator Ty under consideration is a contraction in $C[0,1]$. For this, consider the following quantity:

$$\rho\,(Ty_1, Ty_2) =$$
$$= \max_{0\le x\le 1} |Ty_1 - Ty_2| = \max_{0\le x\le 1} \left| \int_0^1 G(x,s)\,[f(s, y_1(s)) - f(s, y_2(s))]\,ds \right| . \tag{4.1}$$

Assume that the function $f(x, y(x))$ satisfies a Lipschitz condition with respect to its second argument, i.e.,

$$|f\,(x, y_1(x)) - f\,(x, y_2(x))| \le L\,|y_1 - y_2| \tag{4.2}$$

holds, where L is a positive real number.

Substituting (4.2) into (4.1), we obtain the following inequality:

$$\rho\,(Ty_1, Ty_2) \le L \max_{0\le x\le 1} \int_0^1 |G(x,s)|\,|y_1 - y_2|\,ds \le$$

$$\le L \max_{0\le x\le 1} |y_1 - y_2| \max_{0\le x\le 1} \int_0^1 |G(x,s)| ds.$$

Let us evaluate the integral $g(x) = \int_0^1 |G(x,s)|ds$. Since the increasing function $\Phi(t) = \int_0^t z^{-\alpha}(1-z)^{-\beta}\,dz$ satisfies $0 \le \Phi(t) \le \sigma$ for $t \in [0,1]$, applying the triangle inequality to the explicit form (3.7) gives $|G(x,s)| \le |G(1,1)|$ for all $x, s \in [0,1]$, or equivalently,

$$|G(x,s)| \le \left(\frac{|p| + |q|}{|p-q|} + \frac{q^2}{(p-q)^2} \right) \sigma. \tag{4.3}$$

Using inequality (4.3), we derive the following inequality:

$$\rho\,(Ty_1, Ty_2) \le \left(\frac{|p| + |q|}{|p-q|} + \frac{q^2}{(p-q)^2} \right) \sigma L\,\rho\,(y_1, y_2) .$$

If

$$L < \left[\left(\frac{|p| + |q|}{|p-q|} + \frac{q^2}{(p-q)^2} \right) \sigma \right]^{-1},$$

then the operator T is a contraction.

Now we show that $T : C[0,1] \to C[0,1]$.

Let $y \in C[0,1]$. Consider the following inequality:

$$|Ty| = \left| y_0(x) + \int_0^1 G(x,s) f(s, y(s)) ds \right| \le |y_0(x)| + \int_0^1 |G(x,s) f(s, y(s))|\,ds.$$

It is not difficult to show that for the function $y_0(x)$ the inequality

$$|y_0(x)| \le \frac{|A|}{|p-q|} + \frac{|B||q|\sigma}{(p-q)^2} + \frac{|B|}{|p-q|}\sigma$$

holds. Using this inequality and the fact that $f(x, y(x))$ satisfies the Lipschitz condition, we write the following inequality:

$$|Ty| \le \frac{|A|}{|p-q|} + \frac{|B||q|\sigma}{(p-q)^2} + \frac{|B|}{|p-q|}\sigma + L\int_0^1 |y(s)|\,|G(x,s)|\,ds.$$

Using inequality (4.3), we obtain

$$|Ty| \le \frac{|A|}{|p-q|} + \frac{|B||q|\sigma}{(p-q)^2} + \frac{|B|}{|p-q|}\sigma + \left(\frac{|p|+|q|}{|p-q|} + \frac{q^2}{(p-q)^2}\right)\sigma L\int_0^1 |y(s)|\,ds.$$

Denote

$$M = \left(\frac{|p|+|q|}{|p-q|} + \frac{q^2}{(p-q)^2}\right)\sigma, \qquad c_0 = \frac{|A|}{|p-q|} + \frac{|B||q|\sigma}{(p-q)^2} + \frac{|B|}{|p-q|}\sigma.$$

Taking the maximum over $x \in [0,1]$ in the last inequality and using $\int_0^1 |y(s)|\,ds \le \|y\|$, we get $\|Ty\| \le c_0 + ML\,\|y\| < \infty$; in particular, under the contraction condition $ML < 1$ every fixed point satisfies the a priori bound

$$\|y\| \le \frac{c_0}{1-ML}.$$

Moreover, for any $y \in C[0,1]$ the function $f(\cdot, y(\cdot))$ is continuous, hence bounded, on $[0,1]$, and since $G(x,s)$ is continuous on $[0,1]\times[0,1]$, the integral $\int_0^1 G(x,s)f(s,y(s))\,ds$ depends continuously on x. Together with $y_0 \in C[0,1]$ this shows that $Ty \in C[0,1]$, i.e., $T: C[0,1] \to C[0,1]$.

Based on the obtained results, we formulate the following theorem for problem (2.1), (2.2), (2.3).

Theorem 4.1. *If the function $f(x, y(x))$ is continuous, satisfies the condition*

$$|f(x, y_1(x)) - f(x, y_2(x))| \le L\,|y_1 - y_2|,$$

and

$$L < \left[\left(\frac{|p|+|q|}{|p-q|} + \frac{q^2}{(p-q)^2}\right)\sigma\right]^{-1},$$

then problem (2.1), (2.2), (2.3) has a unique solution in $C[0,1]$.

Proof. The proof follows from the contraction mapping principle applied to the operator T in the complete metric space $C[0,1]$. The operator T maps $C[0,1]$ into itself and is a contraction under the given conditions. Therefore, by the contraction mapping principle, T has a unique fixed point in $C[0,1]$, which corresponds to the unique solution of problem (2.1), (2.2), (2.3). □

Example 4.1. Let $\alpha = \beta = \frac{1}{2}$, $p = 2$, $q = 1$ and $A = B = 1$. Then

$$\sigma = \frac{\Gamma(1/2)\,\Gamma(1/2)}{\Gamma(1)} = \pi, \qquad \left(\frac{|p|+|q|}{|p-q|} + \frac{q^2}{(p-q)^2}\right)\sigma = (3+1)\pi = 4\pi,$$

so the smallness condition of Theorem 4.1 becomes $L < \dfrac{1}{4\pi} \approx 0.0796$. Consider the nonlinearity

$$f(x, y) = x^2 + \tfrac{1}{20}\sin y.$$

It is continuous, and since $|\sin y_1 - \sin y_2| \le |y_1 - y_2|$, it satisfies the Lipschitz condition (4.2) with $L = \frac{1}{20} = 0.05 < \dfrac{1}{4\pi}$. By Theorem 4.1, the corresponding problem (2.1)–(2.3) therefore has a unique solution in $C[0, 1]$.

More generally, every f of the form $f(x, y) = g(x) + \lambda\, h(y)$ with $g \in C[0, 1]$ and a globally Lipschitz h (for instance $h = \sin, \arctan, \tanh$, with Lipschitz constant 1) satisfies (4.2) with $L = |\lambda|$, and the hypotheses of Theorem 4.1 hold whenever $|\lambda| < M^{-1}$. This shows that the assumption (4.2) is met by a broad class of nonlinearities and is not unduly restrictive.

5. Conclusion

This paper provides a detailed investigation of a nonlocal boundary value problem for a nonlinear second-order ordinary differential equation with degeneracy at the boundaries of the interval. First, corresponding auxiliary linear problems were solved, the appropriate solution space was specified, and the Green's function was constructed explicitly and from first principles. As a result, the original nonlocal problem was reduced to an equivalent integral equation.

Using the contraction mapping principle, a fundamental result in functional analysis, the existence of a unique solution to the integral equation was proved. The existence and uniqueness of the solution were ensured under a Lipschitz condition and corresponding parametric constraints, and an explicit a priori bound for the solution was obtained. A concrete example illustrates that these conditions are satisfied by a wide range of nonlinear terms.

The obtained results contribute to the development of the theory of nonlinear differential equations and can be applied in the study of practical models involving nonlocal conditions.

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Author's contributions

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