Uzbekistan Journal of Mathematics and Computer Science
Volume-1, Issue-2, Page 25–35 (2025)
DOI: https://doi.org/10.56143/ujmcs.v1i1.4
Research article

Analysis of the dynamics of quadratic mappings of a simplex with skew-symmetric matrices that are not in general position

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Abstract

The Lotka – Volterra systems arise in questions of biology, population genetics, epidemiology, ecology, economics as well as in some branches of theoretical physics, in particular, in solid state physics. Some important questions of ecology (for example, biogens cycles) can be studied using Lotka – Volterra mappings operating in a four-dimensional simplex with homogeneous tournaments. In this regard, the work is devoted to the construction and study of cards of fixed points of Lotka – Volterra mappings operating in a four-dimensional simplex in the case of homogeneous tournaments (for arbitrary coefficients of a skew-symmetric matrix). The card of fixed points gives us a more detailed understanding of the asymptotic behavior of the trajectories of discrete dynamical Lotka – Volterra systems. In the paper, we show that even if the tournaments corresponding to the Lotka – Volterra mappings are homogeneous, among them it is possible to distinguish a class of mappings with skew-symmetric matrices that are not matrices in a general position. It is not possible to generalize this kind of mappings; each of them represents a map of fixed points of a different type. This is clearly noted in the work. It is also shown that even in the case when the tournament corresponding to the Lotka – Volterra mapping is homogeneous, the set of fixed points is infinite and the card of fixed points consists of a convex hull of fixed points belonging to strong faces.

Keywords: *fixed point; homogeneous tournament; quadratic Lotka – Volterra mapping; simplex.*

AMS Subject Classification (2020): *Primary: 37B25 ; Secondary: 37C25; 37C27.*

1. Introduction

One of the main problems in mathematical biology, epidemiology and ecology is the study of the asymptotic behavior of the trajectories of dynamical systems. The works [1-4] are devoted to the study of continuous dynamical systems and the asymptotic behavior of their trajectories. The proposed work is devoted to the analysis of the trajectories of interior points of quadratic Lotka – Volterra mappings operating in a four-dimensional simplex that are not in a general position. Before presenting the main results, let us start with preliminary information and a review of the literature.

Received : *05–june–2025,* ***Accepted :*** *10–november–2025*
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Let

$$S^{m-1} = \{x \in \mathbb{R}^m : x = (x_1, ..., x_m) : x_i \geq 0, \sum_{i=1}^{m} x_i = 1\}$$

the standard simplex in $\mathbb{R}^m$ and $A = (a_{ki})$, $k, i = \overline{1, m}$ – is a skew-symmetric matrix with conditions $|a_{ki}| \leq 1$.

The mapping $V : S^{m-1} \to S^{m-1}$ defined by equality

$$V : x'_k = x_k \left(1 + \sum_{i=1}^{m} a_{ki} x_i\right), \quad k = \overline{1, m}, \tag{1.1}$$

is called the discrete Lotka – Volterra operator. Mappings of the form (1.1) arise in problems of population genetics that describe the evolution of a certain population over time, and time is considered discrete [5].

Each Lotka – Volterra operator and its corresponding skew-symmetric matrix are associated with a complete oriented tournament graph [6], [7] and a partially oriented graph [8].

A complete directed graph – tournament is constructed if the skew-symmetric matrix is in the general position [6]. To build a tournament, let us take m points numbered $1, 2, ..., m$ on the plane and connect the point with the number k to the point with the number i with an arc directed from k to i if $a_{ki} < 0$ and in the opposite direction if $a_{ki} > 0$.

So, the graph constructed in this way is called a tournament corresponding to the Lotka – Volterra operator and we denote it by T_m.

A tournament is called strong if there is a path from any vertex to any other according to the orientation (direction of the arc).

A tournament that does not have strong subtournaments is called a transitive.

Definition 1.1. [9] A tournament is called homogeneous if any of its sub-tournaments is either strong or transitive.

Theorem 1.2. A is a skew-symmetric matrix, then the sets

$$P = \{x \in S^{m-1} : Ax \geq 0\} \quad \text{and} \quad Q = \{x \in S^{m-1} : Ax \leq 0\}$$

non-empty convex polyhedra.

Theorem 1.3. If A is a generic skew-symmetric matrix, then the set P (respectively Q) consists of a single point.

2. The card of fixed points of the operator V

Let us recall the concept of a card of fixed points for a dynamic system (1.1) [9], [10]:

Let $\alpha \subset I = \{1, ..., 5\}$. We represent the set of all fixed points $\{x \in S^4 : Vx = x\}$ of the operator V as points on the plane, then for each $\alpha \subset I$ the fixed point Q_α is connected by an arc to a fixed point P_α directed from P_α to Q_α. The resulting directed graph is called the card of fixed points of the operator V we denote it by G_V.

It is known [11], [12] that for $m = 5$, only the next four tournaments are homogeneous. These are the tournaments shown in Figure 1.

In case a), the tournament is transitive. If the tournament is transitive, then any trajectory of the Lotka – Volterra mapping converges to one of the vertices of the simplex [10]. This means that the fixed point card G_V coincides with the tournament itself T_5. In the case of transitivity, the operator has no fixed points except the vertices of the simplex [12]. Next, we mark the vertices of the tournaments with the numbers $1, 2, 3, 4, 5$ from top to bottom, and the substructure with vertices, for example, $1, 2, 5$, is denoted by $\overline{125}$.

Definition 2.1.[11] A skew-symmetric matrix $A = (a_{ki})$ is called a general position matrix if all major minors of even order are nonzero.

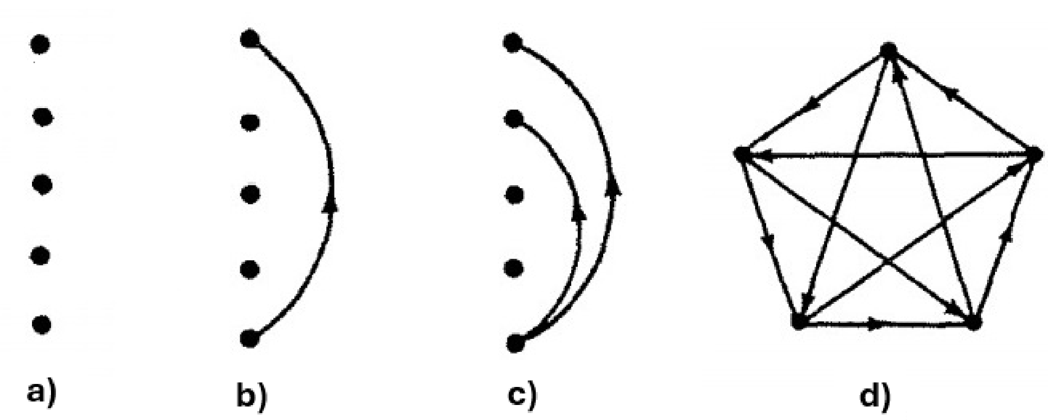

Figure 1. Homogeneous tournament.

If the skew-symmetric matrix of general position, then the corresponding Lotka – Volterra mapping V with coefficients a_{ki} is also a general position operator. The task assigned to us is to study quadratic Lotka – Volterra mappings operating in a four-dimensional simplex that are not in a general position. That is, we show that even if the tournament corresponding to the skew-symmetric matrix is homogeneous, but the matrix itself and, accordingly, the operator may not be in the general position. Since the skew-symmetric matrix of the system is not a matrix of general position, i.e. all major minors of the fourth order (there are only five of them in this case) are zero. Fixed point cards have been constructed and studied for such mappings, since the structure of fixed point cards gives a detailed idea of the asymptotic behavior of the trajectories of interior points of discrete Lotka – Volterra dynamical systems.

In [12], [13] it is proved that skew-symmetric matrices of general position form an open and everywhere dense subset in the set of all skew-symmetric matrices.

For example, the mapping of Lotka-Volterra $V : S^3 \to S^3$ has the form:

$$\begin{cases} x_1' = x_1(1 + a_{12}x_2 - a_{13}x_3 + a_{14}x_4), \\ x_2' = x_2(1 - a_{12}x_1 + a_{23}x_3 - a_{24}x_4), \\ x_3' = x_3(1 + a_{13}x_1 - a_{23}x_2 + a_{34}x_4), \\ x_4' = x_4(1 - a_{14}x_1 + a_{24}x_2 - a_{34}x_3), \end{cases}$$

where $a_{ki} \in [-1; 1],\ k, i = \overline{1, 4}$

This operator is a general position operator if and only if the coefficients $a_{ki} \in [-1; 1],\ k, i = \overline{1, 4}$ satisfy the following conditions:

$$a_{ki} \neq 0,\ k, i = \overline{1, 4} \text{ and } a_{12}a_{34} - a_{13}a_{24} + a_{14}a_{23} \neq 0.$$

Fixed point cards for the Lotka – Volterra operators were first introduced in [5] and it also introduced the concept of a homogeneous card for Lotka – Volterra mappings. Many other useful properties of the fixed point card are given in [12], [13]. But these papers do not consider in detail the cases when the skew-symmetric matrix corresponding to the Lotka – Volterra mapping is not in the general position. Our goal is to consider these cases in more detail, since these mappings can serve as a discrete model of the biogen cycle in an ecosystem. In [14], the Lotka – Volterra mapping is investigated, acting in a four-dimensional simplex as a discrete model of the phosphorus and carbon cycle, depending on the nature of the card of fixed points of this mapping. Here we show that among those operators there can also be those that are not in general position and the set of their fixed points is an infinite set. Let us go over each case in detail.

3. Main results

a) Consider the Lotka – Volterra operator acting in

$$S^4 = \{x = (x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5, x_i \geq 0, \sum_{i=1}^{5} x_i = 1\},$$

with the corresponding transitive tournament T_5.

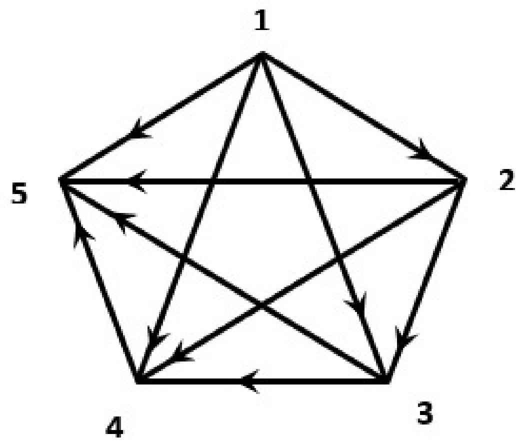

Figure 2. Transitive tournament.

The skew-symmetric matrix corresponding to this operator has the form:

$$A = \begin{pmatrix} 0 & -a_{12} & -a_{13} & -a_{14} & -a_{15} \\ a_{12} & 0 & -a_{23} & -a_{24} & -a_{25} \\ a_{13} & a_{23} & 0 & -a_{34} & -a_{35} \\ a_{14} & a_{24} & a_{34} & 0 & -a_{45} \\ a_{15} & a_{25} & a_{35} & a_{45} & 0 \end{pmatrix}$$

where $|a_{ki}| \leq 1$.

It is easy to see from the classical algebra course that there are only five major minors of the fourth order for this matrix.

$$A_1^{11} = \begin{pmatrix} 0 & -a_{23} & -a_{24} & -a_{25} \\ a_{23} & 0 & -a_{34} & -a_{35} \\ a_{24} & a_{34} & 0 & -a_{45} \\ a_{25} & a_{35} & a_{45} & 0 \end{pmatrix}$$

The determinant of the skew-symmetric matrix A_1^{11} is equal to the following expression:

$$A_1^{11} = \begin{vmatrix} 0 & -a_{23} & -a_{24} & -a_{25} \\ a_{23} & 0 & -a_{34} & -a_{35} \\ a_{24} & a_{34} & 0 & -a_{45} \\ a_{25} & a_{35} & a_{45} & 0 \end{vmatrix} = (a_{23}a_{45} - a_{24}a_{35} + a_{25}a_{34})^2.$$

Similarly, we can calculate the values of the remaining fourth-order minors:

$$A_2^{22} = (a_{15}a_{34} - a_{14}a_{35} + a_{13}a_{45})^2,$$

$$A_3^{33} = (a_{15}a_{24} - a_{14}a_{25} + a_{12}a_{45})^2,$$

$$A_4^{44} = (a_{15}a_{23} - a_{13}a_{25} + a_{12}a_{35})^2,$$

$$A_5^{55} = (a_{14}a_{23} - a_{13}a_{24} + a_{12}a_{34})^2.$$

Now we can select the elements of the skew-symmetric matrix so that the values of these minors are zero,

$$a_{12} = a_{13} = a_{14} = a_{15} = a_{23} = a_{34} = a_{45} = \frac{1}{3}, \quad a_{24} = a_{35} = \frac{2}{3}, \quad a_{25} = 1$$

i.e.

$$A_1^{11} = (a_{23}a_{45} - a_{24}a_{35} + a_{25}a_{34})^2 = (1 - 4 + 3)^2 = 0$$

$$A_2^{22} = (a_{15}a_{34} - a_{14}a_{35} + a_{13}a_{45})^2 = (1 - 2 + 1)^2 = 0,$$

$$A_3^{33} = (a_{15}a_{24} - a_{14}a_{25} + a_{12}a_{45})^2 = (2 - 3 + 1)^2 = 0,$$

$$A_4^{44} = (a_{15}a_{23} - a_{13}a_{25} + a_{12}a_{35})^2 = (1 - 3 + 2)^2 = 0,$$

$$A_5^{55} = (a_{14}a_{23} - a_{13}a_{24} + a_{12}a_{34})^2 = (1 - 2 + 1)^2 = 0.$$

The picture is clear here, since the tournament is transitive, the card of fixed points completely coincides with it.

The tournament shown in Figure is strong and in its expanded form looks as shown in Figure 3.

From the Figure 3 we see that T_5 has three cyclic triples $\overline{125}, \overline{135}, \overline{145}$, i.e. three strong substructures with three vertices. It is known [], [] that if a tournament with three vertices is strong, then the mapping corresponding to this tournament has a fixed point inside the simplex, unlike its vertices. Below we will find the coordinates of these points.

The skew-symmetric matrix corresponding to this strong tournament has the form:

$$A = \begin{pmatrix} 0 & -a_{12} & -a_{13} & -a_{14} & a_{15} \\ a_{12} & 0 & -a_{23} & -a_{24} & -a_{25} \\ a_{13} & a_{23} & 0 & -a_{34} & -a_{35} \\ a_{14} & a_{24} & a_{34} & 0 & -a_{45} \\ -a_{15} & a_{25} & a_{35} & a_{45} & 0 \end{pmatrix}$$

In [], the same operator was investigated when it is in a general position and it is proposed as a discrete model of the carbon and phosphorus cycle in an ecosystem, depending on the type of fixed point map. But as it turned out, for this operator, too, the elements of the skew-symmetric matrix can be selected so that all major minors of the fourth order are equal to zero,

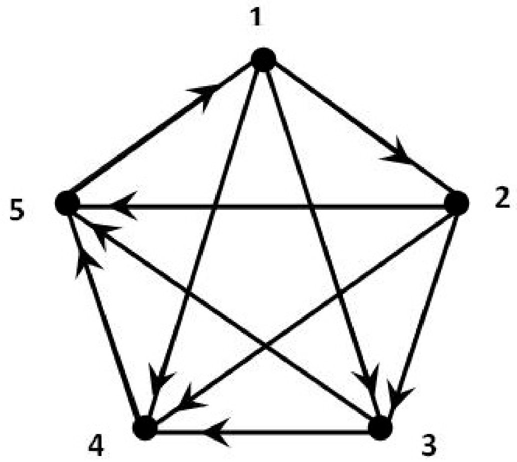

Figure 3. Strong tournament.

$$a_{14} = a_{15} = a_{23} = a_{25} = a_{34} = a_{35} = a_{45} = \frac{1}{3}, \;\; a_{13} = a_{24} = \frac{2}{3}, \;\; a_{12} = 1$$

$$\begin{aligned} A_1^{11} &= (a_{23}a_{45} - a_{24}a_{35} + a_{25}a_{34})^2 = (1-2+1)^2 = 0 \\ A_2^{22} &= (a_{14}a_{35} - a_{13}a_{45} + a_{15}a_{34})^2 = (1-2+1)^2 = 0, \\ A_3^{33} &= (a_{14}a_{25} - a_{12}a_{45} + a_{15}a_{24})^2 = (1-3+2)^2 = 0, \\ A_4^{44} &= (a_{13}a_{25} - a_{12}a_{35} + a_{15}a_{23})^2 = (2-3+1)^2 = 0, \\ A_5^{55} &= (a_{12}a_{34} - a_{13}a_{24} + a_{14}a_{23})^2 = (1-4+3)^2 = 0. \end{aligned}$$

The mapping in this case looks like

$$\begin{cases} x_1' = x_1(1 - x_2 - \frac{2}{3}x_3 - \frac{1}{3}x_4 + \frac{1}{3}x_5), \\ x_2' = x_2(1 + x_1 - \frac{1}{3}x_3 - \frac{2}{3}x_4 - \frac{1}{3}x_5), \\ x_3' = x_3(1 + \frac{2}{3}x_1 + \frac{1}{3}x_2 - \frac{1}{3}x_4 - \frac{1}{3}x_5), \\ x_4' = x_4(1 + \frac{1}{3}x_1 + \frac{2}{3}x_2 + \frac{1}{3}x_3 - \frac{1}{3}x_5), \\ x_5' = x_5(1 - \frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 + \frac{1}{3}x_4). \end{cases} \tag{3.1}$$

and it is not in the general position, since all major minors of the fourth order are zero and the card of fixed points for this operator has the form shown in Figure 4.

The card of fixed points has the form of an undirected graph, which means that the set of fixed points is infinite and consists of a convex hull of three fixed points that belong to the strong faces of the simplex $\Gamma_{125}, \Gamma_{135}, \Gamma_{145}$. Now, in order to investigate the characters of fixed points belonging to the convex hull of fixed points belonging to strong faces, we first find their coordinates explicitly by solving the equation $Vx = x$, according to [10]:

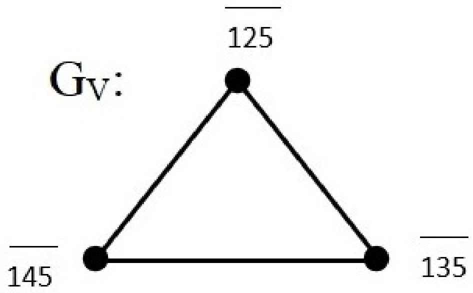

Figure 4. The card of fixed point for homogeneous tournament.

$$M_1\left(\frac{1}{5},\frac{1}{5},0,0,\frac{3}{5}\right),\quad M_2\left(\frac{1}{4},0,\frac{1}{4},0,\frac{1}{2}\right),\quad M_3\left(\frac{1}{3},0,0,\frac{1}{3},\frac{1}{3}\right)$$

Now let us take their convex hull:

$$M_1\left(\frac{1}{5},\frac{1}{5},0,0,\frac{3}{5}\right) \mid \alpha$$

$$M_2\left(\frac{1}{4},0,\frac{1}{4},0,\frac{1}{2}\right) \mid \beta$$

$$M_3\left(\frac{1}{3},0,0,\frac{1}{3},\frac{1}{3}\right) \mid \gamma$$

For the considered mapping, an arbitrary fixed point belonging to this shell has coordinates of the form:

$$M\left(\frac{1}{5}\alpha+\frac{1}{4}\beta+\frac{1}{3}\gamma;\ \frac{1}{5}\alpha;\ \frac{1}{4}\beta;\ \frac{1}{3}\gamma;\ \frac{3}{5}\alpha+\frac{1}{2}\beta+\frac{1}{3}\gamma\right),\ 0\le\alpha,\beta,\gamma\le 1$$

Let $\alpha=\beta=\gamma=\frac{1}{3}$, then the fixed point has the form $M\left(\frac{47}{180},\frac{1}{15},\frac{1}{12},\frac{1}{9},\frac{43}{90}\right)$.

Now we calculate the eigenvalues for this fixed point, i.e. we analyze the spectrum of the Jacobian at this point and get the following:

$$\lambda_1=\frac{1}{90}\left(90+i\sqrt{510}\right)$$

$$\lambda_2=\frac{1}{90}\left(90-i\sqrt{510}\right)$$

$$\lambda_3=1,\quad \lambda_4=1,\quad \lambda_5=1.$$

It is easy to see that the modulo eigenvalues are greater than one. This means that the entire convex hull consists of repulsive fixed points. The definitions describing the characters of fixed points are given in [11], [12].

Now, let us move on to the third tournament from Figure 1 (see Figure 5).

This strong tournament, unlike the previous one, has four strong sub-tournaments, with three vertices - $\overline{135},\overline{145},\overline{235},\overline{245}$. Each of these strong triples has one interior fixed point. Let us select the elements of the skew-symmetric matrix

$$a_{12}=a_{14}=a_{24}=a_{25}=a_{34}=a_{35}=a_{45}=1,\quad a_{15}=a_{23}=2,\quad a_{13}=3$$

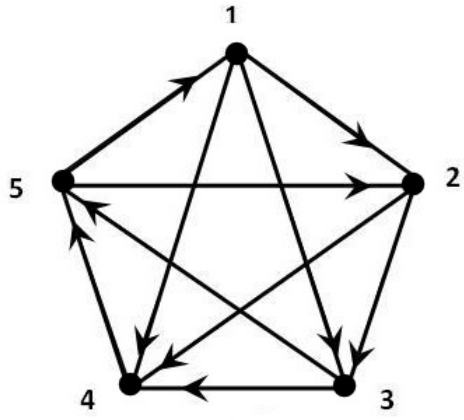

Figure 5. The homogeneous tournament.

and then

$$A = \frac{1}{3}\begin{pmatrix} 0 & -1 & -3 & -1 & 2 \\ 1 & 0 & -2 & -1 & 1 \\ 3 & 2 & 0 & -1 & -1 \\ 1 & 1 & 1 & 0 & -1 \\ -2 & -1 & 1 & 1 & 0 \end{pmatrix}$$

we get the minors equal to zero, i.e.

$$A_1^{11} = (a_{24}a_{35} - a_{23}a_{45} + a_{25}a_{34})^2 = (1-2+1)^2 = 0$$
$$A_2^{22} = (a_{14}a_{35} - a_{13}a_{45} + a_{15}a_{34})^2 = (1-3+2)^2 = 0,$$
$$A_3^{33} = (a_{12}a_{45} - a_{15}a_{24} + a_{14}a_{25})^2 = (1-2+1)^2 = 0,$$
$$A_4^{44} = (a_{12}a_{35} - a_{15}a_{23} + a_{13}a_{25})^2 = (1-4+3)^2 = 0,$$
$$A_5^{55} = (a_{12}a_{34} - a_{13}a_{24} + a_{14}a_{23})^2 = (1-3+2)^2 = 0.$$

Since in this case there are four interior fixed points belonging to strong faces, the card of fixed points looks like a convex hull of them (see Figure 6).

Here, as in the previous case, we can explicitly calculate the coordinates of the vertices of the card and check the characters of the fixed points belonging to this card.

The last – fourth tournament has five strong sub-tournaments, which means that the card has interior fixed points belonging to the faces of the simplex Γ_{124}, Γ_{134}, Γ_{135}, Γ_{235} and Γ_{245}. Here, you can also select the elements of a skew-symmetric matrix, so that all its fourth-order minors are equal to zero.

$$a_{12} = a_{13} = a_{14} = a_{15} = a_{24} = a_{25} = a_{35} = 1, \quad a_{23} = a_{45} = 2, \quad a_{34} = 3$$

$$A = \frac{1}{3}\begin{pmatrix} 0 & 1 & 1 & -1 & -1 \\ -1 & 0 & 2 & 1 & -1 \\ -1 & -2 & 0 & 3 & 1 \\ 1 & -1 & -3 & 0 & 2 \\ 1 & 1 & -1 & -2 & 0 \end{pmatrix}$$

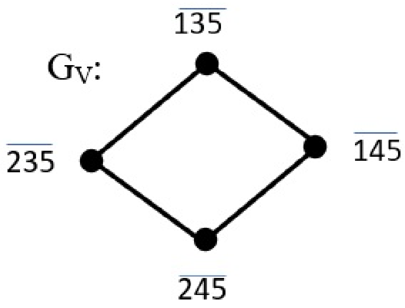

Figure 6. The card of fixed point for homogeneous tournament.

$$
\begin{aligned}
A_1^{11} &= (a_{24}a_{35} - a_{23}a_{45} + a_{25}a_{34})^2 = (1 - 4 + 3)^2 = 0 \\
A_2^{22} &= (a_{13}a_{45} - a_{15}a_{34} + a_{14}a_{35})^2 = (1 + 2 - 3)^2 = 0, \\
A_3^{33} &= (a_{14}a_{25} - a_{12}a_{45} + a_{15}a_{24})^2 = (1 - 2 + 1)^2 = 0, \\
A_4^{44} &= (a_{12}a_{35} - a_{15}a_{23} + a_{13}a_{25})^2 = (1 - 2 + 1)^2 = 0, \\
A_5^{55} &= (a_{13}a_{24} - a_{12}a_{34} + a_{14}a_{23})^2 = (1 - 3 + 2)^2 = 0.
\end{aligned}
$$

The fixed points card of this operator has the form shown in Figure 7.

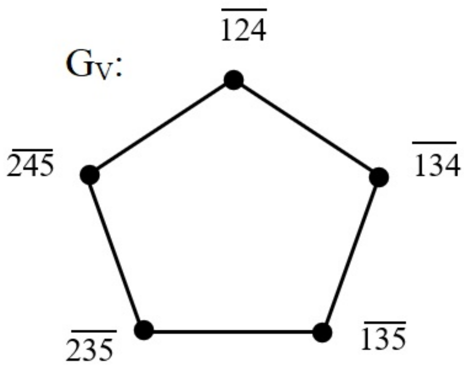

Figure 7. The card of fixed point for homogeneous tournament.

Inonclusion, we have constructively proved the following theorem.

Corollary 3.1. Let give a discrete Lotka – Volterra mapping of the form (1.1). If all the major minors of the second order of the skew-symmetric matrix corresponding to this mapping are nonzero, then

– if all fourth-order minors are nonzero, then all eigenvalues of the skew-symmetric matrix are complex numbers and the kernel is zero,

$$det A \neq 0, \quad Ker A = \{0\},$$

that is, the mapping is in the general position;

– if all the major minors of the fourth order are zero, then the core of the skew-symmetric matrix will be nonzero, i.e. $KerA \neq \{0\}$. The equation $Ax = 0$ has a solution and the eigenvalues of the skew-symmetric matrix are modulo greater than one, which means that the card of fixed points consists of repulsive fixed points.

4. Conclusion

The main result of this paper, in contrast to works [5], [11], [12], is the study of quadratic Lotka – Volterra mappings that are not mappings in general position. Mappings of this nature can be proposed as a discrete model to study the biogen cycle in the ecosystem [13]. In the paper, we analyze the cases where all the principal minors of even order are equal to zero, the set of fixed points is infinite, and the card of fixed points consists of the convex hull of fixed points belonging to strong faces. The main result of the work is Theorem 3, in which the kernel of a skew-symmetric matrix and its eigenvalues are analyzed. As a result, the nature of the fixed points of the considered mappings is determined. The cases considered in this paper can be used as a discrete model of the nitrogen cycle. We will consider the application in the next paper. In the paper we use elements of the graph theory in order to clearly see the dynamic picture of the considered mappings, since the use of elements of graph theory and the construction of cards of fixed points helps to visually build a picture in problems of ecology, epidemiology etc.

Acknowledgements

We, the authors of the paper Analysis of the dynamics of quadratic mappings of a simplex with skew-symmetric matrices that are not in general position would like to thank the Tashkent State Transport University for the opportunity to publish our work.

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