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Research article

The effect of air flow generated by the movement of a high-speed train on the boundary layer

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Abstract

In this paper, we investigate the flow around a high-speed train, both below and above it. Our aim is to ensure the safety of people and objects near high-speed trains by studying the air flow around them. Specifically, we aim to solve the problem of determining the velocity and pressure distributions in air around a moving train in a horizontal plane. We assume that the flow is two-dimensional, potential, and stationary, and use the Zhukovsky's conformal mapping method, the Christopher – Schwartz integral, and Chaplygin's source and sink methods to obtain the velocity field. Based on this velocity field, we calculate pressures and the distance of the air flow from the train's surface. We also determine the force of the air flow on solid particles on a horizontal surface to assess the possibility of their separation from the surface.

Keywords: *Flat Problem, High-Speed Train, Air Flow, Boundary Layer.*

1. Introduction

In railway transport, determining aerodynamic drag and the force of dynamic impact on the train's body are among the most significant problems. In [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11], the problem of airflow around the composition in a plane parallel to the horizon is considered. We will explore this problem in a plane perpendicular to the horizon. Let's consider a plane problem in the horizontal plane. The airflow is flat, potential and stationary.

In Figure 1, the boundaries of the flow domain are represented by $ABFDA$. The solid boundaries are AB and DC, while FD represents the frontal section. FD and DC define the composition of a high-speed train. The free surface FD has an atmospheric pressure. According to the Bernoulli integral (because the air has no weight), the velocity along the surface DA is constant and equal to V_0.

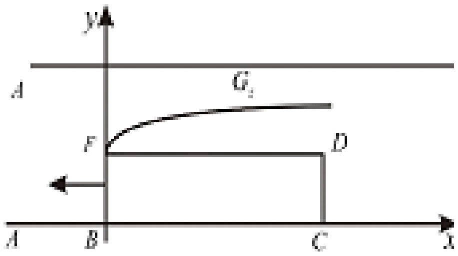

Fig. 1. Semi-infinite body in Cartesian coordinate system. Source: [Compiled by the authors]

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With a comfortable mapping, using the Zhukovsky function, we obtain the coordinates indicated in Fig. 2.

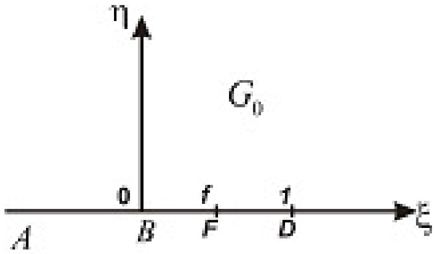

Fig. 2. Semi-infinite body under conformal mapping. Source: [Compiled by the authors]

The pressure in the airflow domain is determined by the following equation:

$$p = p_0 + \rho \frac{V_n^2 - V^2}{2}, \tag{1.1}$$

where V_n is the train speed, $\vec{V}$ is the velocity vector of air particles, $\vec{V} = u\vec{i} + v\vec{j}$, $V = \sqrt{u^2 + v^2}$, u and v denote longitudinal and transverse velocities, respectively.

2. Materials and Methods

Using the Zhukovsky function by a conformal mapping, we study the problem. We use the Christopher – Schwartz integral and Chaplygin's source and sink method to obtain the velocity field function.

In [8] and [9], a method is developed for solving plane and axisymmetric problems related to the potential flow of an ideal, compressible fluid at subsonic velocities.

Introduce the Zhukovsky function

$$\omega(\zeta) = \ln \frac{V_0}{\bar{V}(\zeta)} = \ln \frac{V_0}{V(\zeta)} + i\theta \tag{2.1}$$

where θ is the inclination angle of the velocity vector, V_0, V are the velocity modules on the free surface in the flow domain.

We have the following boundary conditions for the Zhukovsky function in the domain G_0 :

$$\left.\begin{array}{l} \text{Along } AB: \ \eta = 0, \ -\infty < \xi < 0, \ Im\omega = \theta(\xi) = 0; \\ \text{along } BF: \ \eta = 0, \ 0 < \xi < f, \ Im\omega = \theta(\xi) = \frac{\pi}{2}; \\ \text{along } FD: \ \eta = 0, \ f < \xi < 1, \ Re\omega = \theta(\xi) = 0; \\ \text{along } DA: \ \eta = 0, \ 1 < \xi < \infty, \ Im\omega = 0. \end{array}\right\} \tag{2.2}$$

3. Analysis and Results

Introduce the function $\omega_1(\zeta) = \frac{\omega(\zeta)}{\sqrt{\zeta - f}\sqrt{\zeta - 1}}$. $\omega_1(\zeta)$ has the form

$$\omega_1(\zeta) = \frac{1}{2}\int_0^f \frac{dt}{(t-1)\sqrt{(1-t)(t-\zeta)}}.$$

Integrating the last equality, we obtain

$$\omega_1(\zeta) = \frac{1}{2\sqrt{\zeta - f}\sqrt{\zeta - 1}} \ln \sqrt{\frac{\zeta}{\zeta - f}}. \tag{3.1}$$

From here, we obtain the Zhukovsky function in the domain G_0.

For the Zhukovsky function, we have

$$\omega(\zeta) = \ln\sqrt{\frac{\zeta}{\zeta - f}}. \tag{3.2}$$

Then the conjugate complex velocity will be

$$\vec{V} = V_0\sqrt{\frac{\zeta}{\zeta - f}}. \tag{3.3}$$

From the last relation, we obtain the velocity field for air in the domain G_z.

Taking into account that the sources of both inflow and sink are located at points $\xi = 0,\ \eta = 0,\ \xi = 1$, using the Chaplygin source and sink method, we obtain expressions for determining the complex potential in the domain G_0 :

$$\frac{dw}{d\zeta} = \frac{q}{\pi}\frac{1}{1-\zeta}. \tag{3.4}$$

Then we obtain an expression for determining the mapping function $Z(\zeta)$:

$$Z(\zeta) = \frac{q}{\pi\Phi_0}\int_0^{\zeta}\sqrt{\frac{t}{t-f}}\frac{dt}{(1-t)}. \tag{3.5}$$

The velocity V_0 on the free surface DA will be determined from the condition $\lim\limits_{\xi\to\infty,\ \text{at}\ \eta=0}\left[\vec{V}(\xi,\eta)\right] = V_n$ and obtained taking into account equality (3.3).

To integrate equation (3.5), we introduce a variable τ instead ζ in the form: $\tau = \sqrt{\frac{\zeta}{\zeta - f}}$, whence we obtain $\zeta = \frac{\tau^2 f}{\tau^2-1} = f + \frac{f}{\tau^2-1}$, $d\zeta = \frac{-2\tau f}{(\tau^2-1)^2}$.

Integrating differential equation (3.5) with respect to ζ, we get $Z(\zeta) = h_n\int_0^{\zeta}\sqrt{\frac{\zeta}{\zeta-f}}\frac{d\zeta}{1-\zeta}$ where h_n is the wagon width.

The expression for the integrand has the form:

$$I = \int\sqrt{\frac{\zeta}{\zeta - f}}\frac{d\zeta}{1-\zeta} = \int\frac{-2\tau^2 f}{(\tau^2-1)^2}\frac{d\tau}{\left(1 - \frac{\tau^2 f}{(\tau^2-1)}\right)} = \int\frac{-2\tau^2 f}{(\tau^2-1)}\frac{d\tau}{\left[\tau^2(1-f)-1\right]}. \tag{3.6}$$

Expanding the integrand into rational fractions, we have:

$$I = \int\frac{d\tau}{\tau-1} - \int\frac{d\tau}{\tau+1} - \int\frac{d\tau}{\tau\sqrt{1-f}-1} + \int\frac{d\tau}{\tau\sqrt{1-f}+1} =$$

$$= \ln|\tau-1| - \ln|\tau+1| - \frac{1}{\sqrt{1-f}}\int\frac{d\tau}{\tau - \frac{1}{\sqrt{1-f}}} + \frac{1}{\sqrt{1-f}}\int\frac{d\tau}{\tau + \frac{1}{\sqrt{1-f}}} =$$

$$= \ln\left|\frac{\tau-1}{\tau+1}\right| + \frac{1}{\sqrt{1-f}}\ln\left|\frac{\tau + \frac{1}{\sqrt{1-f}}}{\tau - \frac{\tau}{\sqrt{1-f}}}\right| = \ln\left|\frac{\sqrt{\frac{t}{t-f}}-1}{\sqrt{\frac{t}{t-f}}+1}\right| + \frac{1}{\sqrt{1-f}}\ln\left|\frac{\sqrt{\frac{t}{t-f}}\sqrt{1-f}+1}{\sqrt{\frac{t}{t-f}}\sqrt{1-f}-1}\right| =$$

$$= \ln\left|\frac{\sqrt{t}-\sqrt{t-f}}{\sqrt{t}+\sqrt{t-f}}\right| + \frac{1}{\sqrt{1-f}}\ln\left|\frac{\sqrt{t}\sqrt{1-f}+\sqrt{t-f}}{\sqrt{t}\sqrt{1-f}-\sqrt{t-f}}\right|\Bigg|_0^{\xi}.$$

Integrating this equation, we have

$$I_0 = I\left|\begin{matrix}\xi\\0\end{matrix}\right. = \ln\left|\frac{\sqrt{t}-\sqrt{t-f}}{\sqrt{t}+\sqrt{t-f}}\right| + \frac{1}{\sqrt{1-f}}\ln\left|\frac{\sqrt{t}\sqrt{1-f}+\sqrt{t-f}}{\sqrt{t}\sqrt{1-f}-\sqrt{t-f}}\right|\left|\begin{matrix}\xi\\0\end{matrix}\right. .$$

We obtain from here the expression for the mapping function $Z(\zeta)$:

$$\omega_1(\zeta) = \frac{1}{2\sqrt{\zeta-f}\sqrt{\zeta-1}}\ln\sqrt{\frac{\zeta}{\zeta-f}}.$$

$$Z(\xi) = \hat{Z} = \hat{x} + i\hat{y} = \\ = \hat{h}_n\cdot\left(\ln\left|\frac{\sqrt{t}-\sqrt{t-f}}{\sqrt{t}+\sqrt{t-f}}\right| + \frac{1}{\sqrt{1-f}}\ln\left|\frac{\sqrt{t}\sqrt{1-f}+\sqrt{t-f}}{\sqrt{t}\sqrt{1-f}-\sqrt{t-f}}\right|\left|\begin{matrix}\xi\\0\end{matrix}\right.\right). \tag{3.7}$$

This is to define the mapping function $Z(\tau)$ of the domain G_0 onto the domain G_z, where $\tau = \sqrt{\frac{\zeta}{\zeta-f}}$.

Consider the velocity distributions on the side surface FD of the wagon where $h = 0,\ f \le \xi \le 1$. Velocity distribution $u(\xi)$, taking into account equalities (3.1) for the velocity distribution along the side surface of a wagon of a high-speed train

$$V(\xi) = V_n\sqrt{\frac{\xi}{\xi-f}},\ f < \xi < 1, \tag{3.8}$$

where V_n is the train speed.

Equalities (3.3) and (3.4), when taken into account along with equalities (3.7) and (3.8), provide the velocity field on the surface FD of the wagon (refer to Fig. 1).

Let's perform a numerical calculation for $h = 0,\ f + \varepsilon < \xi < 1$. At the acute corner of the car, at the point f, a very high speed occurs. Therefore, in practice, we will conduct an approximate study at points where the angles between the polygonal boundaries are greater than π.

$$f = 0,28; \quad 1 - f = 0,72 \quad \sqrt{1-f} = 0,84853.$$

$$\tau_0 = \sqrt{\frac{\xi}{\xi-f}} = \sqrt{\frac{\xi}{\xi-0,28}}$$

$$\frac{Z}{H}\pi = I,\left(\hat{Z} - i\hat{h}_n\right)\pi.$$

Dividing into real and imaginary parts, we have

$$\left(\hat{Z} - i\hat{h}_n\right)\pi = \frac{(1-\tau)}{(1+\tau)}\frac{(1+f)}{(1-f)}\left(\frac{f+\sqrt{1-f}}{f-\sqrt{1-f}}\right)^{\sqrt{1-f}};$$

$$\left(\hat{Z} - i\hat{h}_n\right)\pi = \ln\left(\frac{(1-\tau)}{(1+\tau)}\frac{(1+f)}{(1-f)}\right) + \ln\left[\left(\frac{\tau+\sqrt{1-f}}{f+\sqrt{1-f}}\frac{f-\sqrt{1-f}}{\tau-\sqrt{1-f}}\right)^{\sqrt{1-f}}\right]$$

or, simplifying, we have

$$\left(\hat{Z} - i\hat{h}_n\right)\pi = \hat{x} + i\hat{y} - i\hat{h}_n\pi = \ln\left(\frac{(1-\tau)}{(1+\tau)}\left(\frac{\tau+\sqrt{1-f}}{\tau-\sqrt{1-f}}\right)^{\sqrt{1-f}}\right) - \\ - \ln\left[\left(\frac{1+f}{1-f}\right)\left(\frac{f-\sqrt{1-f}}{f+\sqrt{1-f}}\right)^{\sqrt{1-f}}\right] \tag{3.9}$$

$$\exp\left[\left(\hat{Z} - i\hat{h}_n\right)\pi\right] = \frac{(1+\tau)}{(1-\tau)}\frac{(1-f)}{(1+f)},$$

$$\exp\left[\left(\hat{Z} - i\hat{h}_n\right)\pi\right] = \left[\frac{(1+\tau)}{(1-\tau)}\left(\frac{\tau+\sqrt{1-f}}{\tau-\sqrt{1-f}}\right)^{\sqrt{1-f}}\right]\cdot C_0 \tag{3.10}$$

where $C_0 = \left(\frac{1+f}{1-f}\right)\left(\frac{\sqrt{1-f}-f}{\sqrt{1-f}+f}\right)^{\sqrt{1-f}}$.

If $f = 0,28$, then $C_0 \cong 0,596$.

The resulting velocity field makes it possible to determine the pressure along DA at the distance $H_0 = \frac{H}{1{,}178} = \frac{4}{1{,}178} = 3,3955$ determined by the formula $p = p_0 - \rho\frac{V^2}{2}$, where H_0 is the distance from the cars.

Using the Delfi program, we obtained the graph shown in Figure 3.

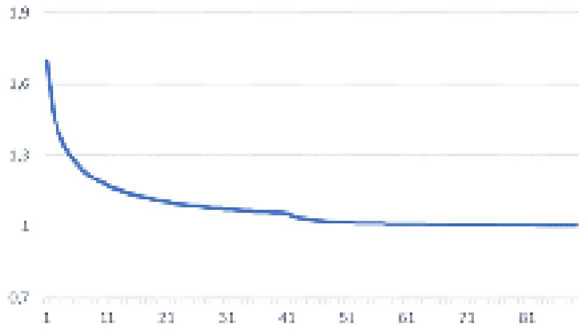

Fig. 3. Air flow velocity during the movement of a high-speed train. Source: [Compiled by the authors]

4. Discussion

Earlier, using the Zhukovsky function, similar to methods, the problem of optics of an aircraft wing with an air flow was solved. In the article of the author entitled «The problem of the flow around the movement of a high-speed train for a flat case» at the international conference «Resource-saving technologies in railway transport» (Tashkent, 2016), this study was discussed.

For the results obtained in this article, the methods considered in the works [1,2,4] were used. In this article, we consider the simplest plane problem in the horizontal plane to de-termine the velocity vector and pressure in the vicinity of a high-speed train.

5. Conclusion

Formulas were derived for calculating the air flow velocity during the movement of a high-speed train using the Zhukovsky method using the Christophel – Schwartz integrals, as well as the distribution of velocities on the side surface of the high-speed train (see Fig. 3). Based on the velocity field obtained, pressures were determined, as well as the distance of the air flow impact formed by the movement of a high-speed train from the side surface of the train.

This study aims to investigate the safety of high-speed trains. We used the results to solve the problem of particles being entrained from the ground by the air flow generated by a high-speed train and also determined the velocity fields for uniform acceleration and deceleration of a high-speed train.

Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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